<u>Logistic Functions</u>

Let a, b, c, and k be positive constants with b<1. A **<u>logistic growth function</u>** is a function that can be written in the form

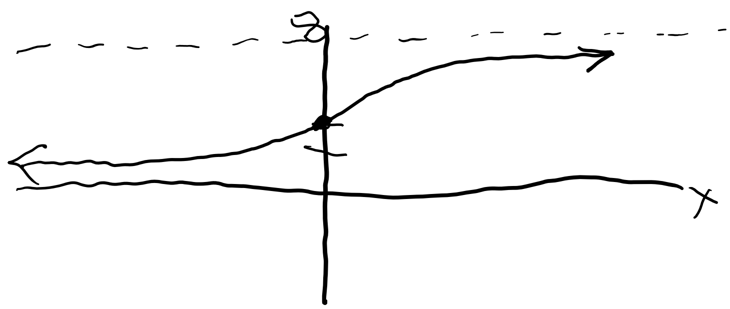
$$f(x) = \frac{c}{1+ab^{x}} \qquad \text{OR} \qquad f(x) = \frac{c}{1+ae^{-kx}}$$

where the constant c is the limit to growth.

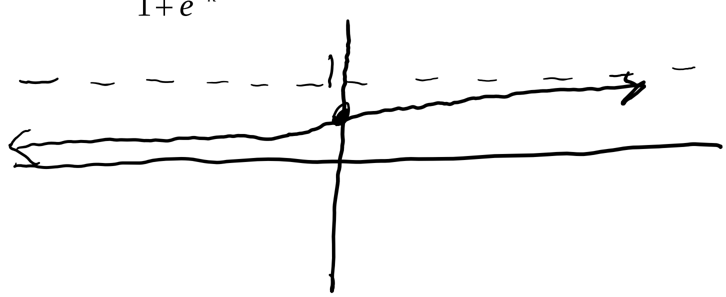
*Logistic Decay: b>1 or k<0

So you will notice that logistic functions have an exponential feel to them; however, that exponential feature is now residing in the denominator and becomes limited by the value in the numerator

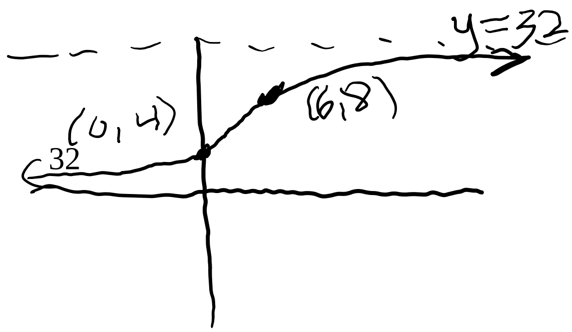
Ex 1) Graph $f(x) = \frac{8}{1+3(0.7)^{x}}$ This logistic function has a limit to growth of 8 (You may have used the term carrying capacity in science class). There has been no vertical shift of this function (a number added or subtracted AFTER the fraction) so there will be horizontal asymptotes at y=0 and y=8. We will find the y-intercept by letting x=0. So, we have a y-intercept of 2. We can now draw using 3 anchors (the two asymptotes and the y-intercept)



Ex 2) Graph $f(x) = \frac{1}{1+e^{-x}}$



Ex 3) Write the equation of the logistic curve graphed for you.



We will use the form $f(x) = \frac{c}{1+ab^x}$

From the picture we know c=32. We need find the values of a and b. We will plug in the intercept point first.

$4 = \frac{32}{1+ab^0}$ $so\ 4 = \frac{32}{1+a}$

$4(1 + a) = 32$

$1 + a = 8$

$a = 7$

Now we find b by plugging in (6,8)

$8 = \frac{32}{1+7b^6}$

$8(1 + 7b^6) = 32$

$1 + 7b^6 = 4$

$b^6 = \frac{3}{7}$

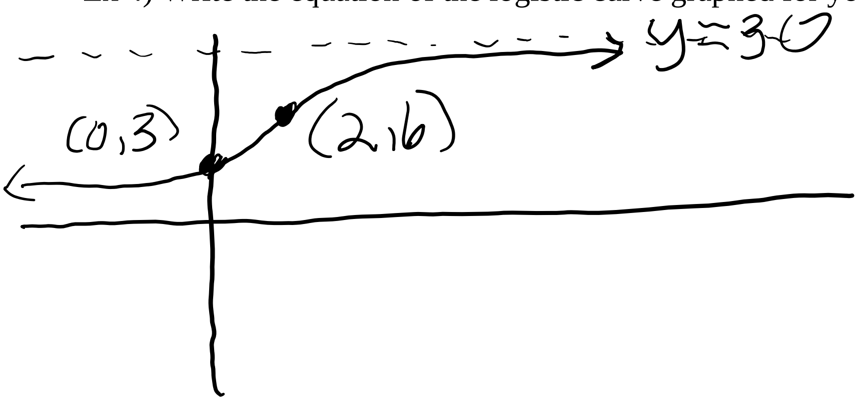
So $b = \sqrt[6]{\frac{3}{7}}$ $or\ b = (\frac{3}{7})^{\frac{1}{6}}$ $or\ b \approx .868$

Equation:

$$f(x) = \frac{32}{1 + 7(.868)^x} \quad or\ in\ exact\ form\ f(x) = \frac{32}{1 + 7(\frac{3}{7})^{\frac{x}{6}}}$$

NOW YOU TRY ONE

Ex 4) Write the equation of the logistic curve graphed for you.



Answer: $f(x) = \frac{30}{1+9(\frac{2}{3})^x}$

Ex 5) after discontinuing all advertising for a certain product in 1994, the manufacturer noted that sales began to drop according to the model

$$S = \frac{500000}{1+0.6e^{kt}}$$

In 1994, t = 0
In 1996, the company sold 300,000 units

a) Complete the model by solving for k
   Use the ordered pair (2, 300000)

   $300000 = \frac{500000}{1+0.6e^{2k}}$

   $300000(1 + 0.6e^{2k}) = 500000$

   $1 + 0.6e^{2k} = \frac{5}{3}$

   $0.6e^{2k} = \frac{2}{3}$

   $e^{2k} = \frac{10}{9} \quad (remember\ 0.6 = \frac{6}{10})$

$2k = ln\frac{10}{9}$

$k = \frac{ln\frac{10}{9}}{2} \approx .0527$

So the complete model would be $S = \frac{500000}{1+0.6e^{.0527t}}$

b) Estimate sales in 1999

Now use the model, plugging in a t value of 5

Answer: $S \approx 280{,}579$