Pre Calculus Lesson
Finance

<u>Compound Interest</u>: Suppose a principal, P, is invested at an annual interest rate, r, compounded k times a year for t years. The amount A in the account after t years is

$$A = P(1 + \frac{r}{k})^{kt}$$

Ex 1) John invested $12,000 at an annual interest rate of 9%. Find the balance after 5 years if the interest is compounded a) quarterly b) monthly

a. $A = 12000(1 + \frac{.09}{4})^{(4*5)}$ $\quad$ $approximately\ \$18,726.11$

b. $A = 12000(1 + \frac{.09}{12})^{(12*5)}$ $\quad$ $approximately\ \$18,788.17$

Ex 2) Lisa has $1500 to invest at a rate of 9.375% interest compounded monthly. How long will it take to double her investment?

$3000 = 1500(1 + \frac{.09375}{12})^{12t}$ ***keep in mind here, we never need a money amount to find doubling time, because the first step will always to divide and end up with 2 on the left hand side of the equal sign.***

$2 = 1.0078125^{12t}$ ***- now change to log form***

$\log_{1.0078125} 2 = 12t$ ***-calculate the log using change of base, then divide 12***

$t \approx 7.4$

Continuous Change Model*:** ***If interest is compounded continuously,

$$A = Pe^{rt}$$

You must see the word continuously to use this formula. It is NEVER inferred.

Ex 3) Tyler wants to invest \$5000 at 7.75% compounded continuously so that he can take care of Mr. Zappia when he retires.

a) ***How much will he have for Zap in 6 years?***

$A = 5000e^{.0775*6}$ $\quad$ $approximately$ $\$7960.07$

b) ***How long will it take for his investment to double (Because Zap isn't getting any younger!)***

$2 = e^{.0775t}$ ***remember doubling just leaves final amount at 2***

Using logs we know $ln2 = .0775t$ ***-calculate ln2 and divide by .0775***

$t \approx 8.9\ years$

<u>Comparing Investments</u>: A common basis for comparing investments is the annual percentage yield (APY). This is the percentage rate that, when compounded annually, would yield the same return as the given interest rate with the given compounding period.

Ex 4) A $2000 investment is made at a bank paying 5.15% annual interest compounded quarterly. What is the equivalent APY? Explain the meaning of your answer.

2000$(1+\frac{r}{1})^{1*1} = 2000(1+\frac{.0515}{4})^{4*1}$ ***so, always assume one year. Use an annual***

rate on the left and the given rate on the right.

So with that in mind, the left side of the equation will always be 1+r

$1+r =$***1.0525***

$r = .0525$ $so\ the\ APY\ is\ 5.25\%$***, meaning to earn the same amount of money as the given quarterly rate, we would need a rate of 5.25% if we were to invest at an annual rate instead.***

To just find an APY for the sake of finding an APY does not help us see the need for it. The idea is if you are comparing investments and they do not use the same compounding period, you can find the APY to take them each back to an annual rate and thus, whichever had the higher percentage would be the better investment. This is what we will do in example 5.

Ex 5) Which is a better investment?

Option A: one that pays 8.75% compounded quarterly or

Option B: one that pays 8.7% compounded monthly

Option A: $1+r = (1+\frac{.0875}{4})^{4*1}$ $-solve\ for\ r.\ APY \approx 9.04\%$

Option B: $1+r = (1+\frac{.087}{12})^{12*1}$ $-solve\ for\ f.\ APY \approx 9.05\%$

Since investment b yields a higher annual rate, it is the better investment

<u>Annuity</u>: A series of equal periodic payments.

<u>Future Value of an Annuity</u>: FV = R $\frac{(1+i)^n - 1}{i}$ ***,***

where R= amount of investment
i= rate per compounding period
n = # of equal periodic payments

Ex 6) At the end of each quarter, Emily makes a $500 payment into a mutual fund. If her investment earns 7.88% compounded quarterly, what will the value be in 20 years?

<u>Present Value</u>: The net amount of money put into an annuity.

$$PV = R\left(\frac{1-(1+i)^{-n}}{i}\right)$$

Ex 7) Joe purchases a car for $18,500. What are the monthly payments for a 4-year loan with $2,000 down if the APR is 2.9%?