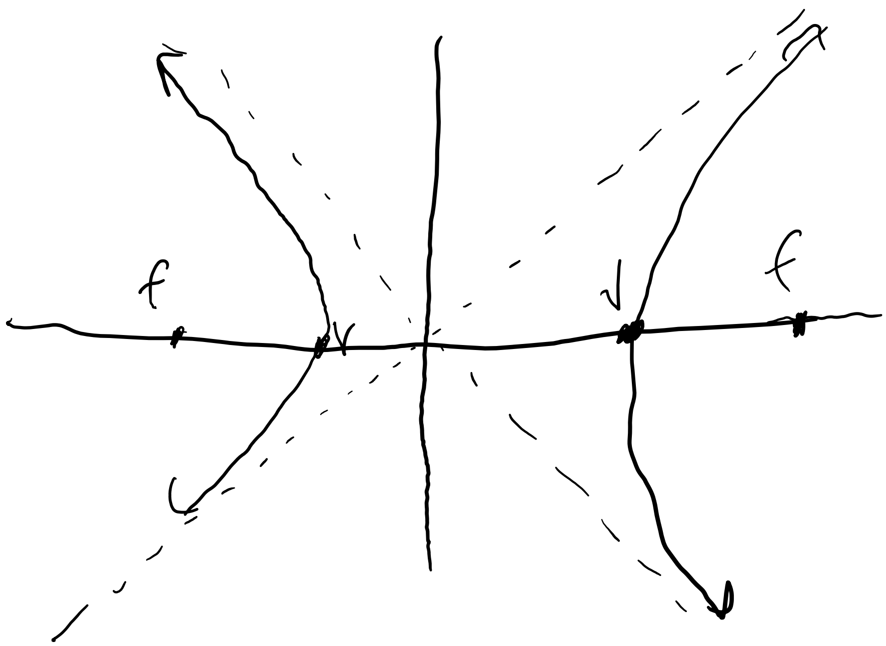
Pre Calculus

<u>Hyperbola</u>: The set of all points in a plane such that the difference of the distances from two fixed points (**<u>foci</u>**) is constant.



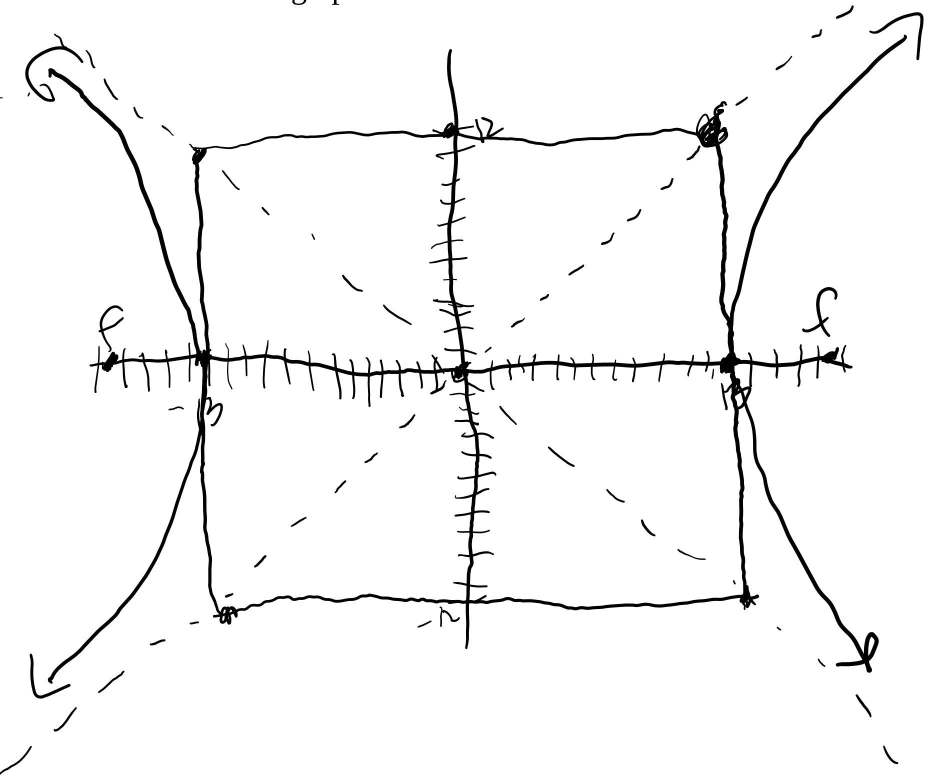
*The chord connecting the vertices is called the **<u>transverse axis</u>**.
*The line perpendicular to the transverse axis with the center as its midpoint is the **<u>conjugate axis</u>**

Standard Form	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Center	(h,k)	(h, k)
	$c^2=a^2+b^2$	$c^2=a^2+b^2$

- a is the distance from center to vertex
- b is the distance from center to conjugate axis endpoint
- c is the distance from center to focus
- Foci always on transverse axis
- Asymptotes always go through the center
- a^2 is always the ***first*** denominator

- Ex) Provide critical information. Then graph.

1. $\frac{x^2}{169} - \frac{y^2}{144} = 1$



Center: (0, 0)

Vertices: $(\pm 13, 0)$

Foci: $(\pm\sqrt{313}, 0)$

CAE: $(0, \pm 12)$

Asymptotes: $y = \pm \frac{12}{13}x$

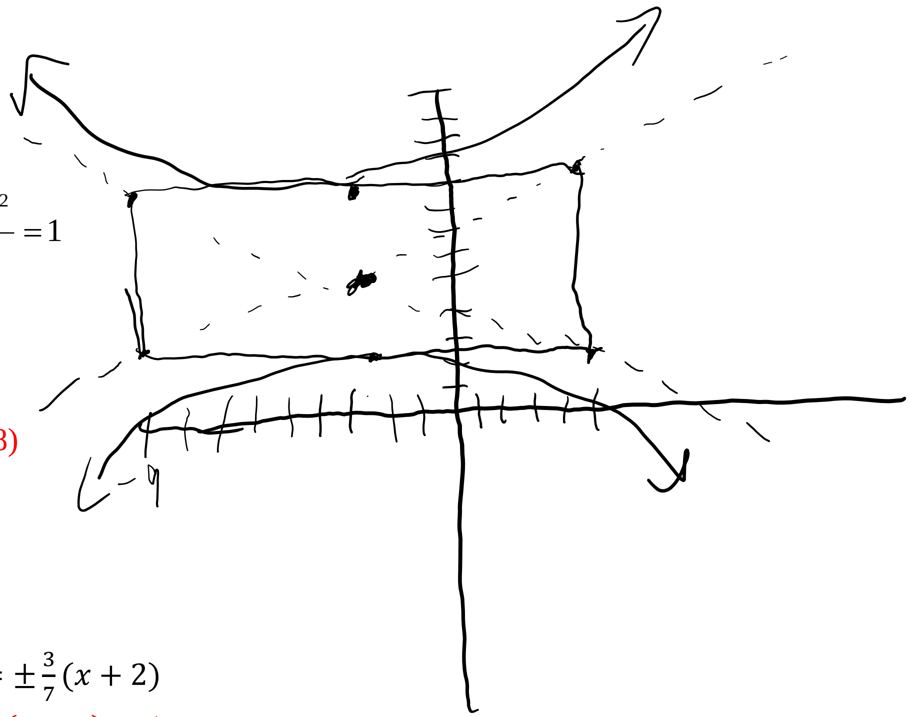
Notes…using a b value of 12, move up and down 12 from each vertex. These are the corner points of the box. Asymptotes are drawn through the center of the hyperbola and the corner points of the box. When drawing the hyperbola, approach the asymptotes and stay out of the box!

Writing Equations of Asymptotes: use point-slope form $y - y_1 = m(x - x_1)$, using the coordinates of the center as (x_1, y_1)

Also notice, since a^2 is no longer the larger denominator, we will now look at numerators to determine how to draw. Since x came first, the hyperbola opens left and right. If the y comes first, the hyperbola will open up and down.

Let's look at one more example

2. $\frac{(y-5)^2}{9} - \frac{(x+2)^2}{49} = 1$



Center $(-2, 5)$

Vertices (-2, 2), (-2, 8)

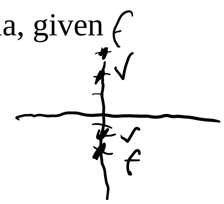
Foci $(-2, 5 \pm \sqrt{58})$

CAE (-9, 5), (5,5)

Asymptotes $y - 5 = \pm\frac{3}{7}(x + 2)$

$y = \pm\frac{3}{7}(x + 2) + 5$

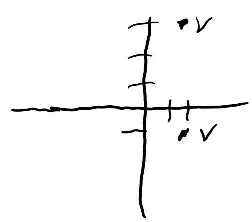
Ex) Write the standard form equation of the hyperbola, given



1. foci (0, ± 3); transverse axis length 4

graphing the foci tells us the center must be (0,0) and c = 3; vertices are on the transverse axis and must be equidistant from the center, which tells us a = 2. Using our focal relationship, $c^2 = a^2 + b^2$, we know that $b^2 = 5$. Since the foci are lining up vertically, we will put the y-values first

answer: $\frac{y^2}{4} - \frac{x^2}{5} = 1$



2. Transverse axis endpoints (2, 3), (2, -1). Conjugate axis length 6

The center would be halfway between vertices (2, 1). The distance from the center to the vertex is a, so a = 2. From the center, we would move 3 to the right and 3 to the left to get conjugate axis endpoints, so b = 3

Answer: $\frac{(y-1)^2}{4} - \frac{(x-2)^2}{9} = 1$

3. $25y^2 - 9x^2 - 50y - 54x - 281 = 0$

$25y^2 - 50y - 9x^2 - 54x = 281$ gather terms to complete the square

$25(y^2 - 2y + 1) - 9(x^2 + 6x + 9) = 281 + 25 - 81$ *notice in this step, we factor out a -9 which changes the sign inside, highlighted in red. Remember we also have to factor out the 25 and the -9 before we complete the square. We now continue*

$25(y-1)^2 - 9(x+3)^2 = 225$

$\frac{(y-1)^2}{9} - \frac{(x+3)^2}{25} = 1$ Standard form must = 1. Divide by 225 on both sides

Now looking in general form, we should be able to identify conic sections

Identifying Conic Sections

General Form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Parabola: A = 0 OR C = 0, B = 0 ***only 1 squared term***

Circle: A = C = 1, B = 0 ***2 squared terms with equal coefficients***

Ellipse: A $\neq$ C, A > 0, C > 0 ***2 squared terms with unequal coefficients, but Same sign***

Hyperbola: A < 0 OR C < 0 ***2 squared terms with different signs***

Ex) Identify the conic without completing the square

1. $2x^2 - y - 8x = 0$ ***Parabola***

2. $x^2 - 8y^2 - x - 2y = 0$ ***Hyperbola***

3. $x^2 + 2y^2 + 4x - 8y + 2 = 0$ ***Ellipse***

4. $3x - 9y^2 + 4y - 9x^2 = 4$ ***Circle***

Homework

Find the vertices and the foci of the hyperbola

1. $\frac{x^2}{16} - \frac{y^2}{7} = 1$
2. $3x^2 - 4y^2 = 12$

Sketch the hyperbola by hand

3. $\frac{x^2}{25} - \frac{y^2}{16} = 1$
4. $\frac{(y-2)^2}{4} - \frac{(x+3)^2}{16} = 1$

Write the standard form equation of the hyperbola

5. Foci $(\pm 3,0)$, transverse axis length 4
6. Transverse axis endpoints (2,3) and (2,-1); conjugate axis length 6
7. $9x^2 - 4y^2 - 36x + 8y - 4 = 0$

Graph the hyperbola. Name the vertices and foci

8. $\frac{(x-2)^2}{4} - \frac{(y+3)^2}{49} = 1$
9. $9y^2 - 4x^2 - 8x - 54y - 4 = 0$