Pre Calculus

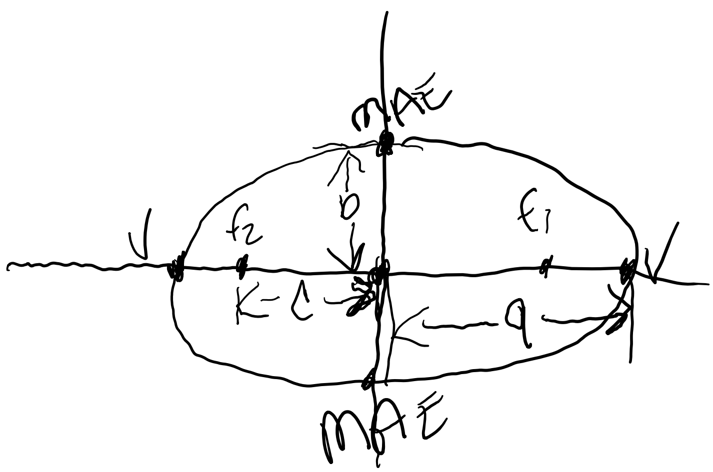
Ellipses

Ellipse: The set of all points in a plane, the sum of whose distances from 2 fixed points is constan

Standard Form:	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
Center:	(h,k)	(h,k)
Major Axis:	Parallel to x-axis	Parallel to y-axis
Minor Axis:	Parallel to y-axis	Parallel to x-axis
Focal relationship:	$c^2=a^2-b^2$	

- a is the distance from the center to the vertex (vertices always on major axis)
- b is the distance from the center to the minor axis endpoint (MAE)
- c is the distance from the center to the focus (foci always on major axis)

Ex)



Ex) Find all critical information and graph

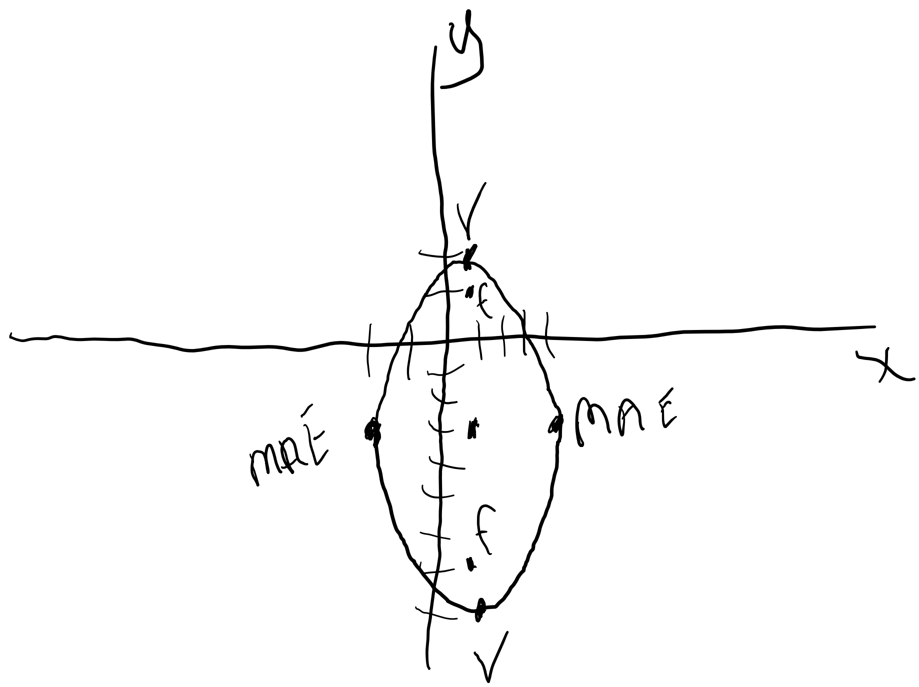
$\frac{(x-1)^2}{9}+\frac{(y+3)^2}{25}=1$

Center: (1, -3)

Vertices (1, 2), (1, -8) *a =5: since larger denominator under y, move up and down 5 to get vertices)*

MAE (-2, -3), (4,-3) *b=3; move right and left 3 to get MAE*

Foci: $c^2=25-9, so\ c^2=16, c=\pm 4$ Foci (1, 1), (1, -7) remember **foci on major axis**



Please note: It cannot be stressed enough, the importance of studying the bullets above. Knowing the purpose and movement as it relates to the values, a, b, c, h, k. It is also important to study these along with the notes from parabolas, so that you are able to distinguish these values in their proper perspectives (parabola or ellipse)

General Form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Ellipse: Ellipses will again have 2 squared terms, but unlike a circle those coefficients will not be equal

They will, however, have the same sign. The presence of a B value would tilt the figure

Again, let's look at all conics in general form perspective

General Form for Conics Circle: 2 squared terms with equal coefficients. Parabola: Only one squared term Ellipse: 2 squared terms with unequal coefficients, but with the same sign.

Ex) Write in Standard form: $3x^2 + 5y^2 - 12x + 30y + 42 = 0$

Completing the square will take a different form. We will not be able to divide by a single number to create a coefficient of 1 on both squared terms. We will therefore factor.

$3x^2 - 12x + 5y^2 + 30y = -42$ gather x and y terms, move constant to other side

$3(x^2 - 4x + \ ) + 5(y^2 + 6y + \ ) = -42$ factor out to create lead coefficient of 1

$3(x^2 - 4x + 4) + 5(y^2 + 6y + 9) = -42 + 12 + 45$ be sure to distribute before adding to other side

$3(x-2)^2 + 5(y+3)^2 = 15$ factor

$\frac{(x-2)^2}{5} + \frac{(y+3)^2}{3} = 1$ To be in standard form we must equal 1 on the right hand side of =

Homework below

Homework

Find the vertices and the foci of the ellipse

1. $\frac{x^2}{16} + \frac{y^2}{7} = 1$
2. $3x^2 + 4y^2 = 12$

Sketch the graph of the ellipse by hand

3. $\frac{y^2}{9} + \frac{x^2}{4} = 1$
4. $\frac{(x+3)^2}{16} + \frac{(y-1)^2}{4} = 1$

Write the standard form equation for the ellipse satisfying the given conditions

5. Foci $(\pm 2{,}0)$, major axis length 10
6. Endpoints of axes $(\pm 4{,}0), (0, \pm 5)$
7. Foci (1, -4) and (5, -4); vertices (0, -4) and (6, -4)
8. Center (2, 3); one vertex (6, 3); one minor axis endpoint (2,6)
9. $9x^2 + 4y^2 - 18x + 8y - 23 = 0$

Graph. Name the center, foci, vertices and minor axis endpoints

10. $\frac{(x+1)^2}{25} + \frac{(y-2)^2}{16} = 1$