Remote Lesson 9.2 Parabolas

Parabola: The set of all points in a plane equidistant from a fixed point (**focus**) and a fixed line (**directrix**)

Note:

- a focus point always sits inside the curve
- a directrix line will never intersect the curve

We will be using the definition and an understanding of focus and directrix to graph parabolas (without a calculator)

Standard Form	$(x-h)^2 = 4p(y-k)$	$(y-k)^2 = 4p(x-h)$
Vertex	(h, k)	(h, k)
Opens	$Up\ if\ 4p > 0$ $Down\ if\ 4p < 0$	$Right\ if\ 4p > 0$ $Left\ if\ 4p < 0$
Axis of Symmetry	$x = h$	$y = k$

- p is the distance from the vertex to the focus (so according to the definition, it is also the distance from the vertex to the directrix). We will use this value to locate those.
- We will use **focal width** (the length of the chord through the focus, perpendicular to the axis of symmetry) to get 2 anchor points on our curve.

Note: **you worked with parabolas in calculator graph ready form, $\boldsymbol{y = a(x-h)^2 + k}$, so that you could put on your calculator and find max, min, zeros, etc. This form does not lend well for left/right opening parabolas and it does not use the geometric advantage we gain from understanding focus and directrix. This is the same figure, just a more graph friendly form.**

Example. Give critical information and graph. $(x+2)^2 = 8(y-3)$

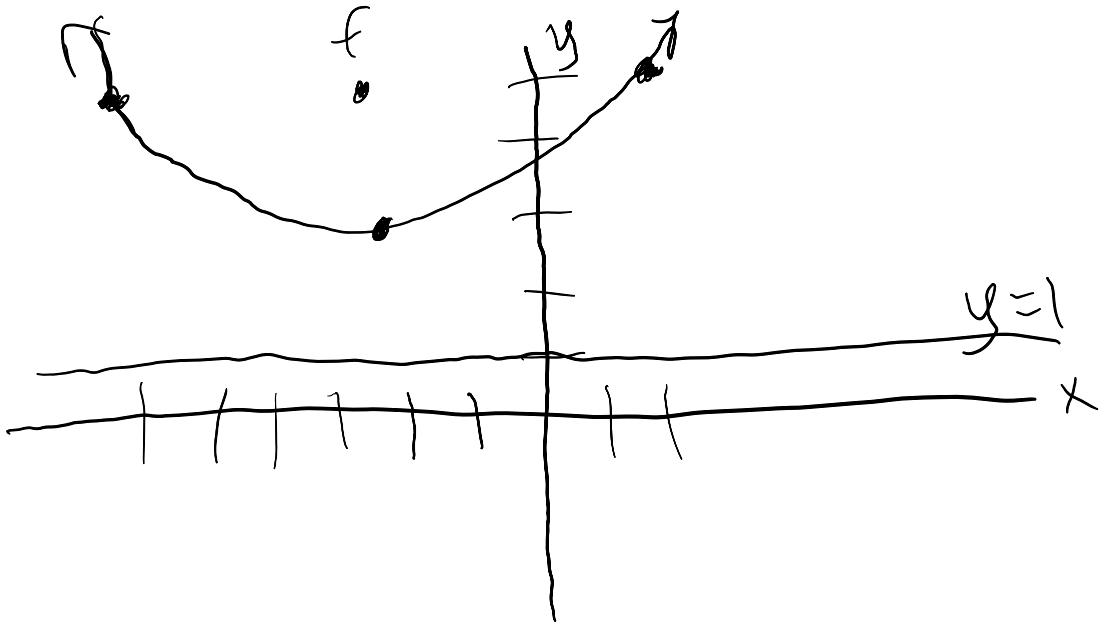
$Vertex\ (-2,3)$

Opens up

$Focus\ 4p = 8, so\ p = 2$. Since a focus point has to be inside the curve, we will move up 2 from the vertex. Focus point is (-2, 5)

Directrix: Again using the p value, we will move down to from the vertex. Since the directrix cannot intersect the curve, it will be a horizontal line. Directrix: y = 1

$Focal\ width$: $By\ definition\ focal\ width = |4p|$, so the focal width is 8. When we graph, we will move 4 to the left and right of the focus and these will be anchor points.



By using our understanding of focus and directrix, if we simply graph the vertex, and then use the p value to move inside the curve we will just need to name an ordered pair for our focus. We can use the same p value to move away from the vertex and figure out the equation of the line. It saves us from having to memorize multiple formulas.

Let's look at another example

Ex. Give critical information and graph $(y-3)^2 = 12x - 24$

We need to be in standard form! $(y-3)^2 = 12(x-2)$

Vertex: (2, 3)

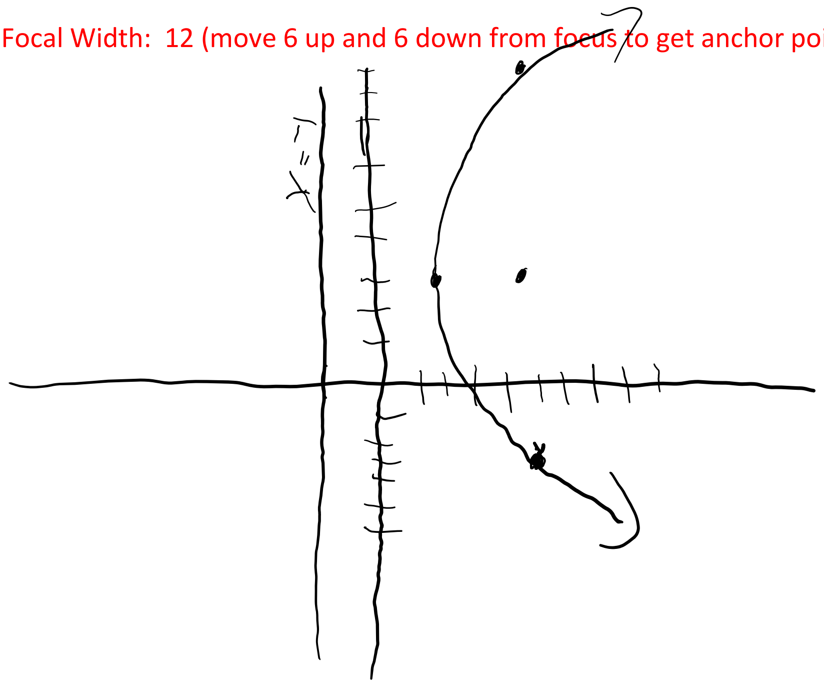
Opens: right (y value is squared)

Focus: p= 3 (graph the vertex and move 3 to the right so focus is inside the curve), Focus (5,3)

Directrix: from the vertex move 3 to the left. Directrix: x= -1

Axis of Symmetry: y = 3

Focal Width: 12 (move 6 up and 6 down from focus to get anchor points)



General Form for Conics: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

To identify a parabola in general form, we need to recognize that parabolas contain **only one squared term**, so either A=0 or C=0.

The B value would indicate a "tilt" of the figure (meaning a slanted axis of symmetry)

So, to summarize general form thus far

<u>General Form Equations</u>

Circle: Two squared terms with equal coefficients

Parabola: Only one squared term

Ex) Give critical info and graph: $y^2 - 10x + 6y + 19 = 0$

To get critical info, we will need standard form, so we will need to complete the square

$y^2 + 6y + 9 = 10x - 19 + 9$

$(y + 3)^2 = 10x - 10$

$(y + 3)^2 = 10(x - 1)$

Vertex: (1, -3)

Opens: right

Focus: 4p =10, so p = 2.5 (we will move right 2.5 from vertex to get focus inside curve)

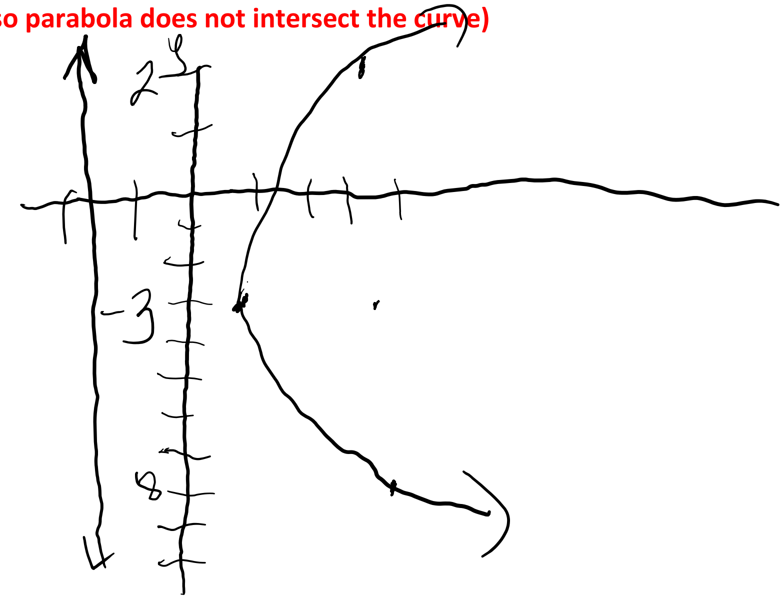
(3.5, -3)

Directrix: (has to be a vertical line so parabola does not intersect the curve)

x = -1.5

Axis of Symmetry: y = -3

Focal width: 10



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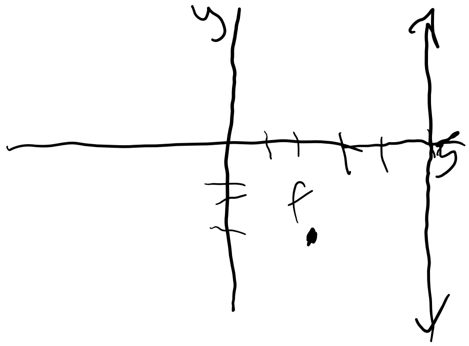
First, graph what you have



the thought: the focus must be inside the curve

the parabola cannot intersect the parabola

we need standard form for a left opening parabola $(y-k)^2 = 4p(x-h)$

the vertex (h, k) by definition is equidistant from focus and directrix

vertex (h, k)= (3.5, -3)

p is the distance from the vertex to the focus. So p = 1.4

Equation: $(y+3)^2 = 4(1.5)(x-3.5)$

$(y+3)^2 = 6(x-3.5)$

HOMEWORK

In 1-5, write the standard form equation of the parabola

1. Vertex (0,0), focus (-3, 0) $y^2 = -12x$
2. Focus (0, 5), directrix y = -5 $x^2 = 20y$
3. Vertex (0, 0), opens downward, focal width=6 $x^2 = -6y$
4. Focus (3, 4), directrix y =1 $(x-3)^2 = 6(y-\frac{5}{2})$
5. Vertex (2, -1), opens upward, focal width=16 $(x-2)^2 = 16(y+1)$

In 6-10 Sketch the parabola and provide all critical information (refer to examples for critical info)

6. $y^2 = -4x$
7. $(x+4)^2 = -12(y+1)$
8. $(y-1)^2 = 8(x+3)$
9. $x^2 + 2x - y + 3 = 0$
10. $y^2 - 4y - 8x + 20 = 0$