Unit 9 Lesson 1: Circles

Circle: The set of all points in a plane at a fixed distance (*radius*) from a fixed point (*center*).

> ***Standard Form Equation for a Circle:*** $(x-h)^2 + (y-k)^2 = r^2$,
>
> center = (h,k) and radius = r

Ex) Write the standard form equation of a circle with center (-2,5) and radius 3

$$(x+2)^2 + (y-5)^2 = 9$$

Be sure to review distance and midpoint formulas for the purpose of writing other equations. Also be able to identify center and radius given a standard form equation.

> ***General Form Equation for All Conics:*** $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
> General form is the expanded form of a standard equation.

If we look at general form, the characteristics that will define an equation of a circle are B=0 and A=C=1.
(In words, there is no xy term in a circle and the coefficients of the squared terms must be equal – including signs!)

Since for circles we will always reduce coefficients of the squares to 1, we will write a special general form for circles (other conics will NOT require this)

> General Form for Circles
>
> $$x^2 + y^2 + Dx + Ey + F = 0$$

To change from general to standard form, we will need to remember how to complete the square.

In the next example, we will change to standard form and review the completing the square process.

Ex.) Find the center and radius of $x^2 + y^2 - 10x + 4y + 17 = 0$

Step 1: Make sure coefficients of squared terms = 1. If not, divide entire equation by the coefficient.

Step 2: Move constant to right hand side of equation

$$x^2 + y^2 - 10x + 4y = -17$$

Step 3: Gather x terms together and y terms together

$$x^2 - 10x + y^2 + 4y = -17$$

Step 4: Multiply one-half times each linear coefficient, square this number and add to both sides of the equation

$$x^2 - 10x + \mathbf{25} + y^2 + 4y + \mathbf{4} = -17 + \mathbf{25} + \mathbf{4}$$

Step 5: Factor the two trinomials and combine like terms on right side of equation.

$$(x-5)^2 + (y+2)^2 = 12 \quad Center(5,-2) \quad radius\ 2\sqrt{3}$$

Ex) Find the center and radius of the circle that passes through the points (-2,3), (6,-5), and (0,7)

Since the points given are not related to a radius or center, we will take each and substitute into general form: $x^2 + y^2 + Dx + Ey + F = 0$

(-2,3): $(-2)^2 + 3^2 - 2D + 3E + F = 0$ $\quad OR \quad -2D + 3E + F = -13$
(6,-5): $6^2 + (-5)^2 + 6D - 5D + F = 0$ $\quad OR \quad 6D - 5E + F = -61$
(0,7): $0 + 7^2 + 0D + 7E + F = 0$ $\quad OR \quad 7E + F = -49$

So now we must solve the system for D,E,F and substitute the values into general form. Then we complete the square to find standard form so that we may name center and radius.

Rather than solving this system, we will use the matrix feature on the TI. We will create a matrix of coefficients and constants on the calculator. It will look like this

$$\begin{bmatrix} -2 & 3 & 1 & -13 \\ 6 & -5 & 1 & -61 \\ 0 & 7 & 1 & -49 \end{bmatrix}$$

This is a 3x4 ("3 by 4") matrix because it has 3 rows and 4 columns. These are the dimensions of the matrix

IN THE TI
2nd x^{-1} (MATRX) Choose EDIT- ENTER
MATRIX A 3x4 (enter dimensions)
A matrix is formed for you to type in entries
(note 1st column is D-coeff, the 2nd column is E-coeff, etc. that is why we have a 0 placeholder in row 3)
When finished typing entries in the matrix, 2nd MODE (QUIT)

Go back into matrix menu, 2nd x^{-1}(MATRX) Choose MATH
On this menu, scroll to B: rref (this stands for row reduced echelon form) ENTER
You should now see rref (on your homescreen.
Go back into matrix menu, 2nd x^{-1}(MATRX) Choose NAMES ENTER so that [A] now has been carried to home screen and now you should see rref([A]) Press ENTER

You should see a matrix that looks like this now

$$\begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -21 \end{bmatrix}$$

Consider the top row with the context we entered it…it would be
1D + 0E + 0F = -10, in other words D = -10. Looking at the next 2 rows the same way
D = -10, E = -4, F = -21. We will now plug these values back into general form.

$x^2 + y^2 - 10x - 4y - 21 = 0$ *From here we complete the square to find center and radius*
$(x^2 - 10x + 25) + (y^2 - 4y + 4) = 21 + 25 + 4$
$(x - 5)^2 + (y - 2)^2 = 50$ *Our equation is in standard form, so we can now answer*
Center (5, 2) Radius $5\sqrt{2}$

The process on the calculator should be reviewed. We could solve any system of equations this way. It is a bit of a long process, but it allows the calculator to use elimination to solve a system of equations.

Lesson 9.1 Homework

I. Write the equation of the circle using the given information

1. Center (3,-4); radius 1.5
2. Endpoints of a diameter (-2,-6), (4, 3)
3. Center (2,4); (-1,3) is a point on the circle
4. Passing through (8,2), (1,9), and (1,1)
5. Passing through (0,-1), (2,-3), (4,-1)
6. Passing through (-3,-2), (-2,-3), (-4,-3)
7. Center (7,-3); tangent to the y-axis

II. Find the center and radius of each circle

1. $x^2 + y^2 - 2y - 15 = 0$
2. $x^2 + y^2 - 8x - 6y + 21 = 0$
3. $4x^2 + 4y^2 - 16x - 9y - 5 = 0$