Remote Lesson 8.3

Exponential and Log Equation Solving.

This lesson is going to be practice solving equations algebraically using the ideas we learned in Lessons 8.1 and 8.2. Geometrically if we ever want to confirm a solution, we could go to the y= screen on the calculator and graph the left side as our first equation. We could then type the right hand side in the 2nd equation and use 2nd trace intersect to find the solution. HOWEVER, that is not the intent of this lesson.

LOG EQUATIONS

1. $\log_2 x + \log_2(x+2) = \log_2(x+6)$ goal- use properties to create one log on each side

   $\log_2(x^2+2x) = \log_2(x+6)$ Log of a sum becomes one log of a product

   $x^2+2x = x+6$ once there is one log on each side, they may be dropped

   $x^2+x-6=0$ it's quadratic, so set it = 0

   $(x+3)(x-2)=0$ factor (use quadratic formula if not factorable!)

   $x=-3, x=2$ set factors = 0 and solve---NOW check answers because logs have restricted domains. We cannot take logs of nonpositive numbers

   x=2 final answer

2. $\log x - \frac{1}{2}\log(x+4) = 1$ There is a constant in this equation (no log). Simplify left side

   $\log x - \log(x+4)^{\frac{1}{2}} = 1$ coefficient of log becomes exponent

   $\log \frac{x}{(x+4)^{\frac{1}{2}}} = 1$ difference of 2 logs becomes log of a quotient

   $10^1 = \frac{x}{(x+4)^{\frac{1}{2}}}$ since one log, change to exponential form

   $10\sqrt{x+4} = x$ get rid of fraction; change exponent to radical

   $100(x+4) = x^2$ square both sides

   $x^2 - 100x - 400 = 0$ quadratic…set it equal to 0

   Quadratic formula: ≈ 103.85

3. $\ln\sqrt{x+2} = 1$ a log only on one side. Change to exponential. Remember, ln has a base of e

$e^1 = \sqrt{x+2}$ now square both sides to solve for x (remember e is a number)

$e^2 = x + 2$

$x = e^2 - 2$ This would be an exact answer

$x \approx 5.389$ this is the approximate value of x

EXPONENTIAL EQUATIONS-The first method of solution for exponential equations is to try to create like bases. If the bases are alike, we can set the exponents equal to each other and solve that way. When this is not possible, we will need to change to log form.

4. $4^x = 32$ both sides can be taken down to a base of 2

$(2^{2)^x} = 2^5$ now, a power raised to a power, we multiply exponents

$2^{2x} = 2^5$ we can now set exponents = and solve for x

$2x = 5$

$x = \frac{5}{2}$ *We do not need to check our answer because the original function is exponential and the domain of an exponential function is all reals.

5. $20(\frac{1}{2})^{\frac{x}{3}} = 5$ 1st step ALWAYS isolate the exponential

$(\frac{1}{2})^{\frac{x}{3}} = \frac{1}{4}$ now create like bases

$(\frac{1}{2})^{\frac{x}{3}} = (\frac{1}{2})^2$ bases are alike, we can now set exponents equal and solve

$\frac{x}{3} = 2$

$x = 6$

6. $3^x = 17$ we clearly cannot make the bases the same to arrive at an exact answer. We will use Logs to solve. 2 methods of solution will be offered. Please follow both.

Method 1: Change the given statement into log form

$\log_3 17 = x$ Knowing we must approximate this answer, use the change of base formula

$\frac{\log 17}{\log 3} = x$ This will allow us to approximate using our calculator

$x \approx 2.579$

Method 2: We are allowed to apply a log to both sides of the equation

$\log 3^x = \log 17$ we will now use the property $\log x^c = c \log x$

$x \log 3 = \log 17$ we will now isolate x

$x = \frac{\log 17}{\log 3}$

$x \approx 2.579$

Method 2 IS the algebraic way of solving this equation. Notice in line 3 we arrive at the same place the change of base formula took us. As we discussed in class, the idea of a formula was to create a shortcut for a repetitively used process. Method 2 is where the Change of Base formula came from! You are welcome to use either method as you go forth solving…whichever you do more efficiently.

7. $5(2)^y = 185$ remember we ALWAYS isolate the exponential first

$2^y = 37$ we cannot get like bases, so use logs (either method)

$\log_2 37 = y$	or	$\log 2^y = \log 37$
$\frac{\log 37}{\log 2}$ = y		$y \log 2 = \log 37$ $y = \frac{\log 37}{\log 2}$
$y \approx 5.209$		$y \approx 5.209$

8. $e^x - e^{-x} = 10$ first remember e is a number≈ 2.71828

Though logs could be applied to both sides, we cannot distribute logs, so it would look like this $\log(e^x - e^{-x}) = log10$. While this is ok, there is no way to move forward with it. So we will use our knowledge of exponentials to "clean it up" a little first

$e^x - \frac{1}{e^x} = 10$ first we get rid of the negative exponent (review this if needed)

$e^x\left(e^x - \frac{1}{e^x}\right) = (10)e^x$ next we multiply both sides by e^x to get rid of the fraction

$e^{2x} - 1 = 10e^x$ seeing the 2x in the exponent, our equation is taking a "Quadratic" form

We will move everything to one side and set =0

$e^{2x} - 10e^x - 1 = 0$ Factoring does not look possible and with e being a number it is hard to decide how to plug into quadratic formula. So we are going to create a substitution to make the problem manageable. We will always go to the linear term at this point, disregarding coefficient, so we will let $u = e^x$. (keep in mind if $e^x = u, then\ e^{2x} = u^2$ (laws of exponents)

$u^2 - 10u - 1 = 0$ This is our new equation using those substitutions. We must use Quadratic formula here to solve.

$u = \frac{10 \pm \sqrt{100-(-4)}}{2}$

$u = \frac{10 \pm \sqrt{104}}{2}$ If you are comfortable simplifying radicals, and it would help the process, I would recommend it, but it is not necessary.

We now need to put back our original value, x.

$e^x = \frac{10+\sqrt{104}}{2}$, $e^x = \frac{10-\sqrt{104}}{2}$ remember, we let $u = e^x$

We can now change to log form (ln makes sense here, since our base is e)

$\ln\frac{10+\sqrt{104}}{2} = x$ $\ln\frac{10-\sqrt{104}}{2} = x$

$x \approx 2.312$ x gets no answer here as the value of the fraction is negative and we cannot take logs of negatives.

These problems are drawn out I know, but please study the process, especially substituting to make problems look more manageable

Let's try one more

9. $\frac{2^x+2^{-x}}{2} = 3$ get rid of fraction

$2^x + 2^{-x} = 6$ get rid of negative exponent

$2^x + \frac{1}{2^x} = 6$ get rid of fraction again

$2^x\left(2^x + \frac{1}{2^x}\right) = (6)2^x$ BE CAREFUL IN THE NEXT STEP! $2^x(2^x) = 2^{x+x} = 2^{2x}$

$6(2)^x = 6(2)^x$ cannot simplify

$2^{2x} + 1 = 6(2)^x$ set equal to 0

$2^{2x} - 6(2)^x + 1 = 0$ create substitution using "linear term". Disregard coefficient

$let\ u = 2^x$ substitute values into above equation

$u^2 - 6u + 1 = 0$ non-factorable- use quadratic formula

$u = \frac{-6\pm\sqrt{32}}{2}$ now that we solved for u, go back to x

$2^x = \frac{-6+\sqrt{32}}{2}$ $\qquad$ $2^x = \frac{-6-\sqrt{32}}{2}$ recall we had let $u = 2^x$

Change to log form (either common or natural will work here)

$\log_2 \frac{-6+\sqrt{32}}{2} = x$ $\qquad$ $\log_2 \frac{-6-\sqrt{32}}{2} = x$ Use change of base formula. Careful plugging in!

$x \approx 2.543$ $\qquad$ $x \approx -2.543$

HW p.331 1-17 odd; 25-37 odd