Remote Lesson 8.2
Logarithmic Functions

Analyze the graph of the log function f(x) = log x

Domain: $(0, \infty)$

Range *All reals*

Increasing *throughout domain*

Decreasing *never*

Continuity *Continuous*

Boundedness *not bounded*

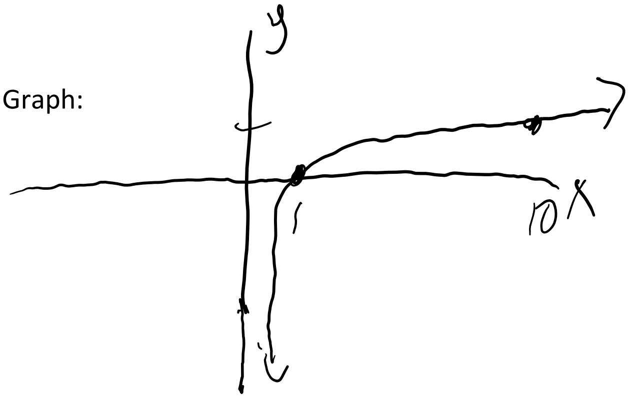
Symmetry *none*

Asymptotes $x = 0$

Extrema *none*

End Behavior $\lim_{x \to \infty} f(x) = \infty, \lim_{x \to -\infty} f(x) = dne$

Graph:



***** $y = log_b x \; iff \; b^y = x$ You MUST be able to quickly and efficiently change from one form to another!!**

Examples

1. Write in log form
   a. $5^2 = 25$ $\quad log_5 25 = 2$
   b. $4^0 = 1$ $\quad log_4 1 = 0$
   c. $8^{-1} = \frac{1}{8}$ $\quad log_8 \frac{1}{8} = -1$

2. Write in exponential form
   a. $\log_4 16 = 2$ $\quad 4^2 = 16$
   b. $\log_7 \frac{1}{49} = -2$ $\quad 7^{-2} = \frac{1}{49}$
   c. $\log_{12} 1 = 0$ $\quad 12^0 = 1$ ***notice, this statement would mean anytime we take the log of 1, The answer will be 0, regardless of the base of the log*

3. Evaluate *(for each expression, begin by attaching = x to the statement)*

   a. $log_3 9$ =2

   b. $log_4 32$ $4^x = 32$ in order to solve for x, we need to get like bases
   $2^{2x} = 2^5$
   So, $x = \frac{5}{2}$

   c. $log_5 \sqrt{5}$ $5^x = \sqrt{5}$
   $5^x = 5^{\frac{1}{2}}$ you must now how to change radicals to exponents!
   $x = \frac{1}{2}$

   d. $6^{log_6 4}$ *This is an exponential statement. We will change to log form*
   $log_6 x = log_6 4$
   So x = 4

Definition

Common Logarithm: Log with base 10 *(calculator does common logs)*. This base is not written.

$\therefore$ log 10 =1 $(because\ 10^x = 10, so\ x = 1)$

log 100 =2

log 1 =0

log 0.1=-1

so log 27.3≈ 1.4362 means $10^{1.4362} \approx 27.3$

Recall $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ = e (y = e^x--the natural exponential function)

Its inverse is a natural logarithm (ln). $\therefore y = \ln x\ iff\ e^y = x$

Examples: Evaluate

1. $\ln \sqrt{e} = \frac{1}{2}$ $(\ because\ e^x = \sqrt{e}, or\ e^x = e^{\frac{1}{2}})$
2. $\ln e^5$ = 5 $(because\ e^x = e^5, so\ x = 5$

Graphing: Ln and Log graph the same (just different bases). Know the transformations

Example) How is the graph of $f(x) = \log x$ transformed to obtain the graph of

1. $f(x) = \log(x-2)$ H shift R2
2. $f(x) = \log(-2x+4)$ H shrink $\frac{1}{2}$, reflect over y-axis, H shift R2
3. $f(x) = -\log x + 6$ Reflect over x-axis, V shift up 6
4. $f(x) = 3logx - 4$ V stretch 3, V shift down 4

IT IS **CRITICAL** THAT YOU KNOW LOG PROPERTIES WELL!

<u>Properties of Logarithms (also works for Ln)</u>

1. $log_b(rs) = log_b r + log_b s$ $(\ln(rs) = \ln r + \ln s)$
2. $log_b \frac{r}{s} = log_b r - log_b s$ $(\ln \frac{r}{s} = \ln r - \ln s)$
3. $log_b r^c = c * log_b r$ $(\ln r^c = c \ln r)$

<u>Change of Base Formula</u>

$$log_b a = \frac{log_c a}{log_c b}$$

***Smart bases to change to would be 10 or e as both can be done on the calculator.

So to put $\log_3 19$ into the calculator, we would apply the formula and get

$\log_3 19 = \frac{\log 19}{\log 3} \approx 2.6801$. Notice if we had changed to natural logs, $\log_3 19 = \frac{\ln 19}{\ln 3} \approx 2.6801$

HW>> p. 308 (1-47 odd) p. 317 (1-35 odd 39, 41, 53)