Pre Calculus

Unit 8 Lesson 1 Exponential Function Review

Recall from the toolkit—exponential functions are of the form $\boldsymbol{f(x) = ab^x}$

Compare and contrast the analyses of the following two graphs

$f(x) = 2^x \quad (note\ b > 1)$

Domain $(-\infty, \infty)$

Range $y > 0$

Increasing throughout domain

Decreasing never

Symmetry none

Boundedness bounded below

Continuity continuous

Extrema none

Asymptotes $y = 0$

End Behavior $\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = 0$

$f(x) = (\frac{1}{2})^x \quad (note\ 0 < b < 1)$

Domain $(-\infty, \infty)$

Range $y > 0$

Increasing never

Decreasing throughout domain

Symmetry none

Boundedness below

Continuity continuous

Extrema none

Asymptotes $y = 0$

End Behavior $\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to-\infty} f(x) = \infty$

Compare/Contrast the analyses. **Don't forget to graph!**

Describe how the graph of $f(x) = 2^x$ is transformed to obtain the given function

Anchor points for $f(x) = 2^x$ would be (0,1), (1,2), (-1, $\frac{1}{2}$)

1. $g(x) = 2^{x+1}$ horizontal shift left 1
2. $h(x) = 2^{-x}$ reflected over y-axis
3. $k(x) = -3(2^x)$ vertical stretch of 3; reflect over x-axis

Make sure you can draw the above examples without a calculator

Given the function $f(x) = (1 + \frac{1}{x})^x$, complete the table and try to find the limit of the function as x approaches infinity

x	1	10	100	1000	10,000	100,000	1,000,000
$(1 + \frac{1}{x})^x$	2	2.593	2.705	2.717	2.718	2.71827	2.71828

Notice the values of x are approaching infinity, like a limit…the y-value is the actual limit of the function.

Definition $\lim_{x\to\infty}(1 + \frac{1}{x})^x = \mathrm{e} \approx 2.71828 \ldots$

$f(x) = e^x$ is referred to as **<u>the Natural Exponential Function</u>**. Its graph would sit somewhere between $\boldsymbol{f(x) = 2^x}$ ***and*** $\boldsymbol{f(x) = 3^x}$ *Graph these 3 functions on your calculator*

Recall: When solving exponential equations, the first goal, if possible is to get like bases

Ex) Solve

$$\boldsymbol{3^{x-1} = \frac{1}{27}}$$

Create like bases: $3^{x-1} = 3^{-3}$

Once alike, set exponents equal and solve: $x - 1 = -3 \ldots .so\ x = -2$

This should be familiar and should be practiced so we can move on.

Writing Exponential Equations

Exponential equations are equations of the form $f(x) = ab^x$

- *a* represents the initial amount
- *b* represents rate of growth/decay

  $b = 1 + r, if\ r\ is\ a\ growth\ rate$

  $b = 1 - r, if\ r\ is\ a\ decay\ rate$
- *x* will typically represent time (but could also be other quantities)
- *f(x)* represents final amount

Example- Write the equation for the exponential function passing through the points (0,4) and (2, 36).

$f(x) = ab^x$

$4 = ab^0$ (plug in the ordered pair)

$a = 4$

$36 = 4b^2$(plug in a and the 2nd ordered pair)

$9 = b^2$

b = 3 (principal (positive) value)

Final answer $f(x) = 4 \cdot 3^x$

Example Write the exponential function given

a. Initial value 52, increasing at a rate of 2.3% per day $f(x) = 52(1.023)^x$

b. Initial value 5, decreasing at a rate of 0.59% per week $f(x) = 5(.9941)^x$

   First, remember to change 0.59% to a decimal, we back up the decimal point 2 places (.0059)

   Then, we remember it's decay (b=1-r)

c. Initial value 250, doubling every 7.5 hours. $f(x) = 250(2)^{\frac{x}{7.5}}$

   If the doubling period had been every hour, the exponent would simply be x, but here we have to break those hours into 7.5 hour intervals

Example: The population of Bridgetown is growing at a rate of 2.5% per year. The present population is 50,000. If this trend continues, what will the population be in 5 years?

$f(x) = ab^x$

$f(x) = 50000(1.025)^5$

$f(x) \approx 56570.411$

Please remember you are now answering about how many people, so round to the nearest whole unit.

Final answer: 56, 570

Example: The population of New Zealand in 1985 was 3,295,000 with an annual average growth rate of 1.4%. Assume this trend continues indefinitely.

a.) Express the population p as a function of n, the number of years after 1985
$P(n) = 3295000(1.014)^n$

b.) Predict the population for 1986, 1987, 2007, and 2020
1986- 3341130, 1987- 3387906, 2007- 4473943, 2020- 5360235

If you were to look on Google, the current population of New Zealand is 4,822,233. As you learned in science classes, no population could sustain such dramatic growth. This is why exponential models are good to discuss short term growth, but in the long term, there will be limiting factors that stop this from happening.

Example

POPULATION OF PHOENIX (IN THOUSANDS)

1950	107
1960	439
1970	584
1980	790
1990	983
2000	1,321

Use the data in the table and exponential regression to find a model for the population of Phoenix. Predict the population for 2007 using this model. (Use 1900 as t = 0)

Please remember when given a table of information, you need to do a regression on your calculator.

STAT EDIT

L_1	L_2
50	107
60	439
70	584
80	790
90	983
100	1,321

2nd Y= , turn on plots

ZOOM 9: Statistics (make sure you have cleared equations from y=)

STAT CALC 0:ExpReg

Model: $\boldsymbol{f(x) \approx 20.840(1.045)^x}$ –keeping decimals here for calculating is ok

To predict for 2007, we want more than the 3 decimals our answer uses.

Y=

VARS 5:Statistics

EQ 1:RegEq

ENTER

GRAPH

Remember you may have to change the window to fit the x-value we are asking for (in this example 107)

2nd TRACE

CALCULATE 1: value

Type in x=107

$y \approx 2231.304$

Remember this is the number of thousands (according to the chart)

Final Answer: 2231.304(1000)= 2,231,304

HW Exponential Functions: p. 287 7-37 odd , p. 331 1-6 all

Exponential Models: p. 296 3-21 odd, 29, 31