Remote Lesson 7.5

The purpose of this lesson is taking what we know about intercepts and asymptotes that we learned in Lesson 7.4 (might be helpful to have those notes with you as you go through this one!) and coming up with a sketch of the graph (for the purpose of analyzing as we did in Unit 6)

Ex) Graph and analyze: $f(x) = \frac{2x^2-9x-5}{x^2-3x-28}$

We would first write this function in factored form: $f(x) = \frac{(2x+1)(x-5)}{(x-7)(x+4)}$

We would look through the analysis list and fill in the parts we can without the need for a graph

Domain: $(-\infty,-4) \cup (-4,7) \cup (7,\infty)$

Range: $y > 2, y < 0.513$

Inc/dec: Inc $(-\infty,-4) \cup (4,2.509)$

Dec $(2.509, 7) \cup (7,\infty)$

Continuity: Essential disc x= -4, x=7

Boundedness: not bounded

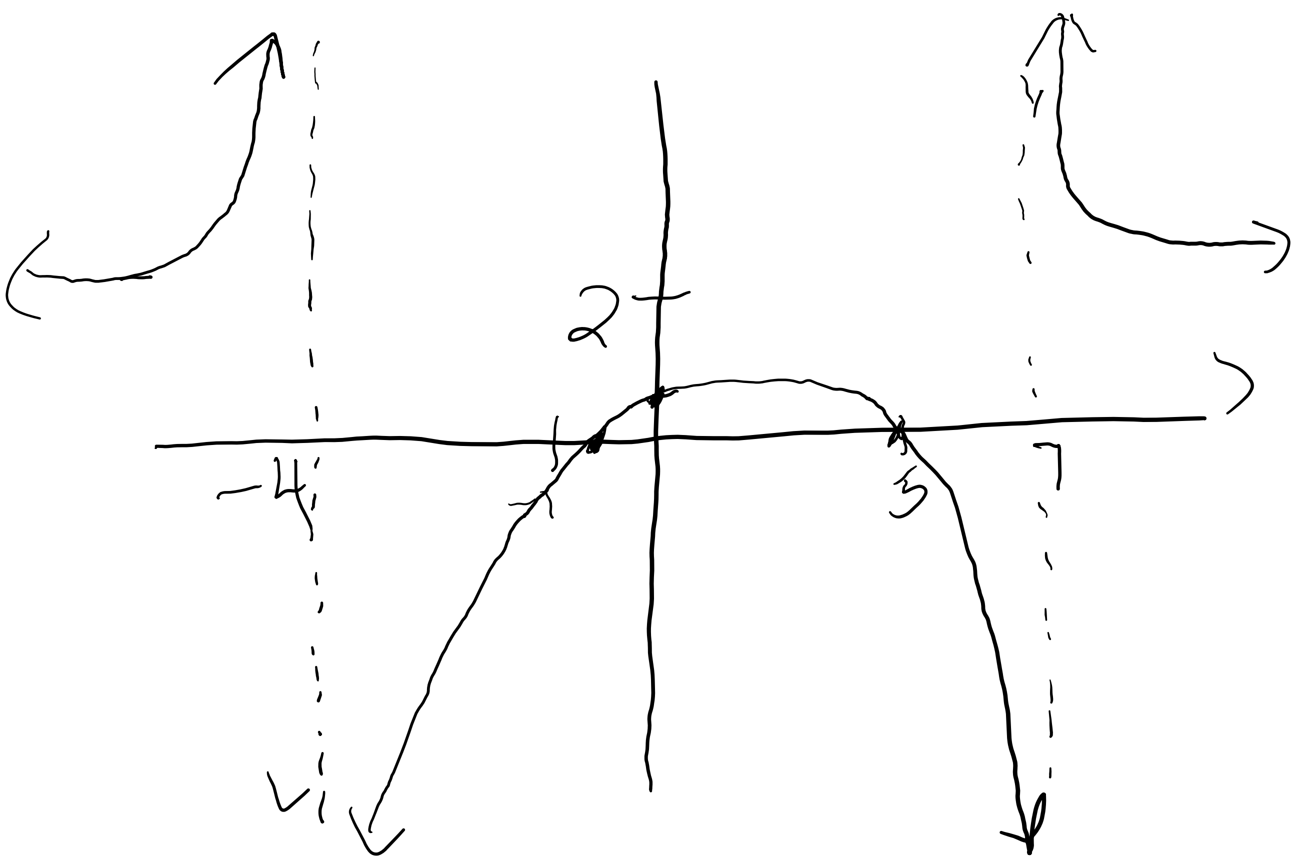
Symmetry: no symmetry

Extrema: rel max 0.513

Asymptotes: V: x= -4, x= 7; EBA: y=2

End Behavior: $\lim_{x\to\infty} f(x) = 2 \quad \lim_{x\to-\infty} f(x) = 2$

Now we will find intercepts and start to graph… $x - int\ -\frac{1}{2}, 5 \quad y - int\ \frac{5}{28}$. *Graph asymptotes as well*



You may use a calculator to find the maximum point, which will be needed for several more items in the analysis (the parts gained after the graph are in RED in the above analysis)

Notice in this example why we have changed the term to EBA instead of horizontal asymptote. In the middle of this function we crossed the horizontal line y=0. Hence the change

Ex) Graph and analyze $f(x) = \frac{x-2}{x^2-4x-12}$

$f(x) = \frac{x+3}{(x-6)(x+2)}$

Domain: $(-\infty,-2) \cup (-2,6) \cup (6,\infty)$

Range: all reals

Inc: never

Dec: throughout domain

Continuity: essential disc x= -2, x=6

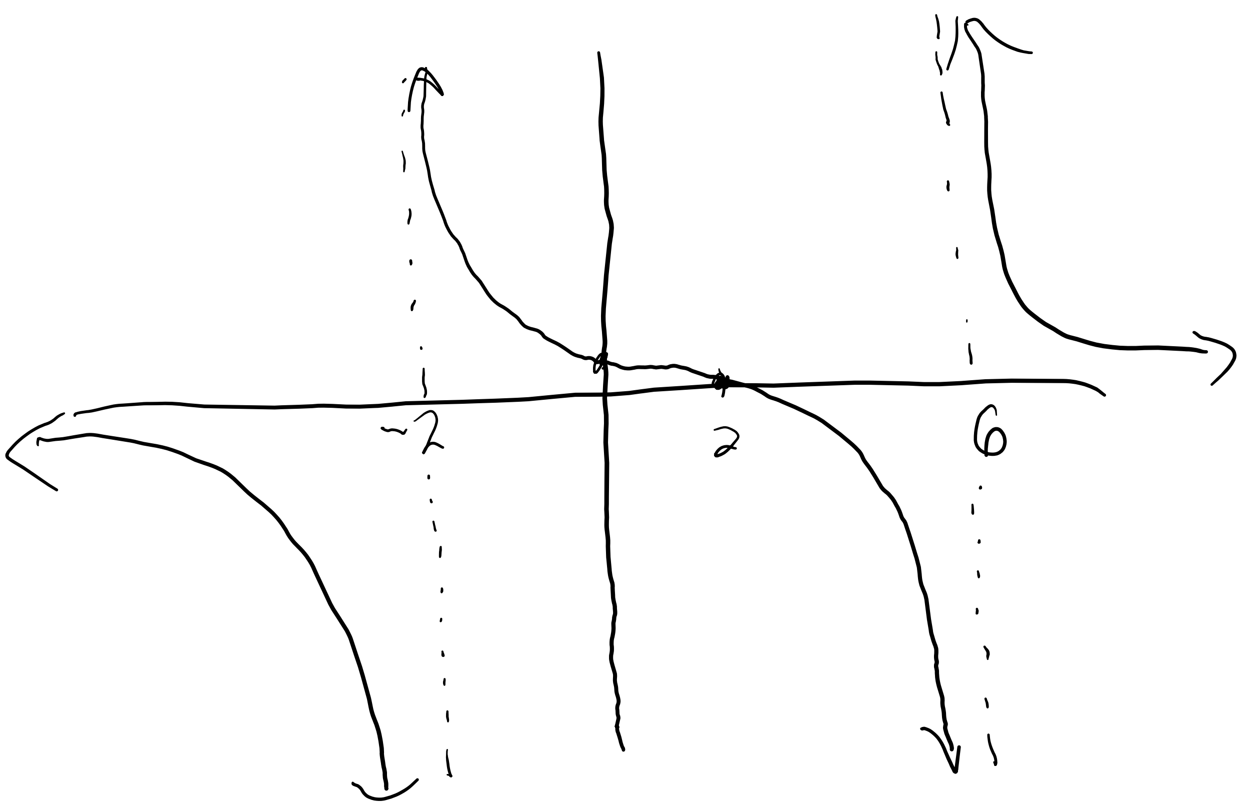
Boundedness: Not bounded

Symmetry: No symmetry

Extrema : none

Asymptotes: V x=-2, x=6, EBA: y=0

End Behavior: $\lim_{x\to\infty} f(x) = 0$ $\lim_{x\to-\infty} f(x) = 0$



Please keep the notes on Asymptotes in mind throughout. In the first example, the degree of the numerator was equal to the degree of the denominator, so we divide the leading terms to get the EBA. In the second example, the degree of the numerator is less than the degree of the denominator, so the EBA is y=0. If you look at the graph in example 2, the function actually crosses the EBA. That is why we changed the name from horizontal asymptote to end behavior asymptotes, because at the ends of the function it will become an asymptote.

One more example

Graph and analyze $f(x) = \frac{x^2-4}{x^3-x^2-6x}$ (this should be done without a calculator)

So, we would first look at our function in factored form: $f(x) = \frac{(x+2)(x-2)}{x(x+2)(x-3)}$

In the analysis, without looking at the graph, we should be able to fill in domain, continuity, asymptotes, end behavior

Domain: $(-\infty,-2) \cup (-2,0) \cup (0,3) \cup (3,\infty)$

Extrema : none

Range: $y \neq \frac{-2}{5}$

Asymptotes: V x=0, x=3, EBA: y=0 (deg N<deg D)

Inc: never

End Behavior: $\lim_{x\to\infty} f(x) = 0$ $\lim_{x\to-\infty} f(x) = 0$

Dec: throughout domain

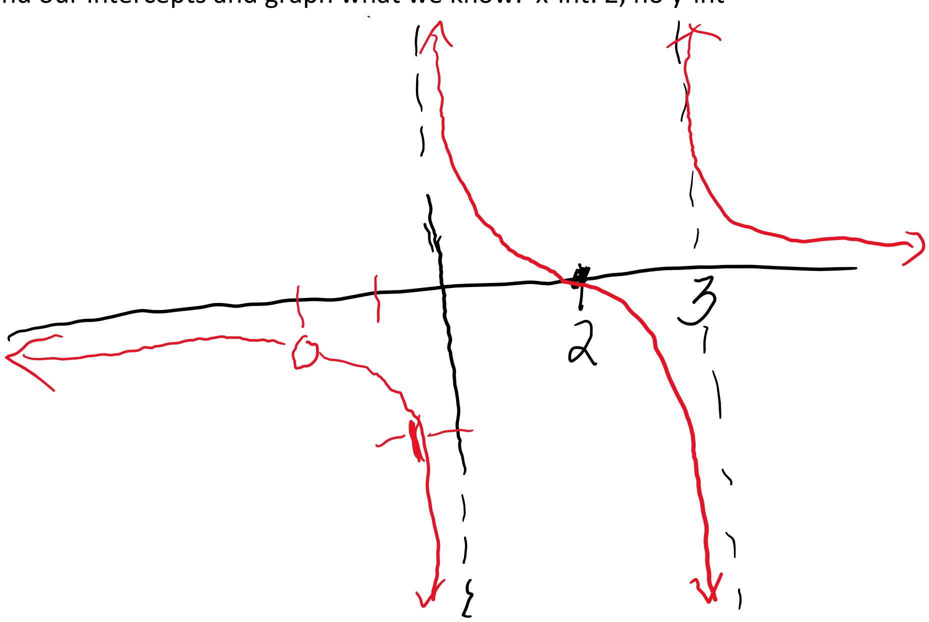
Continuity: essential disc x= 0, x=3

Removable disc x = -2 (x+2) is a removable factor, this will create a hole

Boundedness: not bounded

Symmetry: no symmetry

We will find our intercepts and graph what we know: x-int: 2, no y-int



Remember, an x-intercept is a zero…our zero (x=2) has odd multiplicity, so we know our function will cross the axis at this point. Choose a value between 0 and 2 and plug it into the factored form to see if our answer is pos or neg. If x=1, $f(x) = \frac{(1+2)(1-2)}{1(1+2)(1-3)} = \frac{(+)(-)}{(+)(+)(-)} = (+)$, so we know when x=1, the function is above the x-axis. We may now draw the part of the function in the interval (0,3)

Next, we need to find out where to put the hole. To do this we will remove the discontinuity, and then plug that value into the remaining function. $f(x) = \frac{(x-2)}{x(x+3)}$, and now plug in -2, the location of the hole.

$$f(-2) = \frac{(-2-2)}{-2(-2-3)} = \frac{-4}{10} = \frac{-2}{5}$$

Given that this is a rational function and we know what the ends should look like, we are ready to draw and complete the analysis (done above, answered in red)

It should be noted that if the degree of the numerator is less than or equal to the degree of the denominator, the sketch should be attempted without graphing on the calculator. If the degree of the numerator is greater than the degree of the denominator, graph it on your calculator 1st and then complete the analysis.

HW: Graph and analyze the following functions.

1. $f(x) = \frac{x^2-x-2}{x^2-2x-8}$

2. $f(x) = \frac{x-1}{x^2-x-12}$

3. $f(x) = \frac{4x^2+2x}{x^2-4x+8}$

4. $f(x) = \frac{x^3-2x^2+x-1}{2x-1}$