Remote Lesson 7.3

Task 1: Absolute Value Equality and Inequality

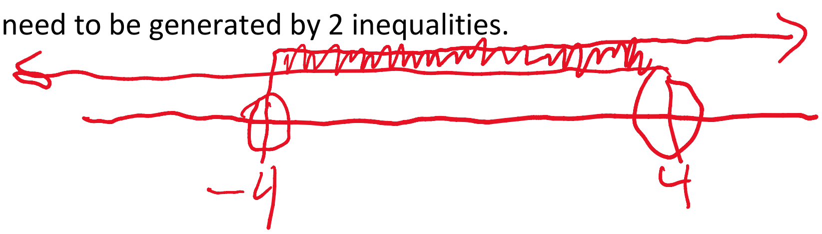
1. Write the two equations you would use to clear the absolute value sign and solve $|\boldsymbol{x-1}| = \boldsymbol{4}$
   Circle the connecting word (**AND** / **OR**) that you would place between the two equations.
   [Hint: **AND** is used when a solution must satisfy both equations. **OR** is used when a solution must satisfy at least one of the equations.]

   The quantity inside the absolute value bars could either equal 4 or negative 4, so that when we take the absolute value we will arrive at a solution of 4.
   So, $x - 1 = 4 \; or \; x - 1 = -4$

   What are the solutions? Solve the above equations $x = 5 \; or \; x = -3$. Check your answers

2. Write the two inequalities that you would use to clear the absolute value sign and solve $|\boldsymbol{x-1}| < 4$
   Circle the connecting word (**AND** / **OR**) that you would place between the two inequalities.

   Think about what it means for a quantity (x-1, not just x) to be less than 4. For absolute value, this is going to need to be generated by 2 inequalities.



Looking at the number line, this is why we use "AND" because the only numbers whose absolute values are less than 4 are in the common area of the 2 graphs. Now think back to our quantity—it was x-1. So our inequalities are $x - 1 < 4 \; and \; x - 1 > -4$.

Observe the statements, the first was made by removing the absolute value bars. The less than sign tells us to use AND. We then remove the absolute value, turn the inequality, and make the right side negative. (this could also apply to the equation in #1).
Solving our inequalities $x < 5 \; and \; x > -3$

Represent the solution using:

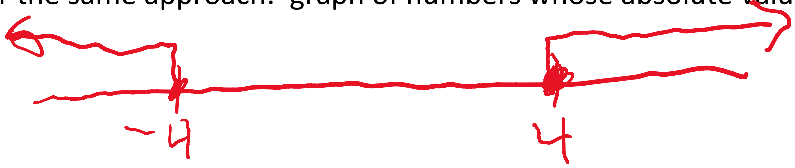
A number line: 

Interval notation: (-3,5)

An inequality(s): $-3 < x < 5$

3. Write the two inequalities that you would use to clear the absolute value sign and solve $|\boldsymbol{x}-\mathbf{1}| \geq \mathbf{4}$
Circle the connecting word (**AND** / **OR**) that you would place between the two inequalities.

Consider the same approach: graph of numbers whose absolute value is greater than or equal to 4



The graph indicates the areas where absolute value is greater than or equal to 4. So, consider our quantity and create inequalities.

$x-1 \geq 4 \; or \; x-1 \leq -4$

Look at the inequalities as compared to the original problem. Just as in the last problem, we removed the absolute values and wrote the first inequality. We then removed the absolute values, turned the inequality, and changed the sign on the number on the right hand side. Now solve: $x \geq 5 \; or \; x \leq -3$

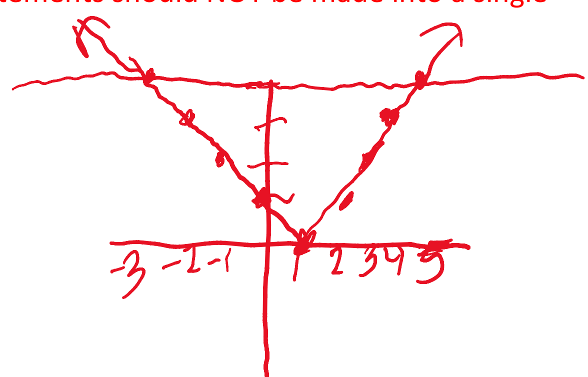
Represent the solution using:

A number line:

Interval notation: $(-\infty, -3] \cup [5, \infty)$

An inequality(s): $x \geq 5 \; or \; x \leq -3$ (Note: "or" statements should NOT be made into a single inequality!)

4. Sketch the graphs of $\boldsymbol{y} = |\boldsymbol{x}-\mathbf{1}|$ and $\boldsymbol{y} = \mathbf{4}$



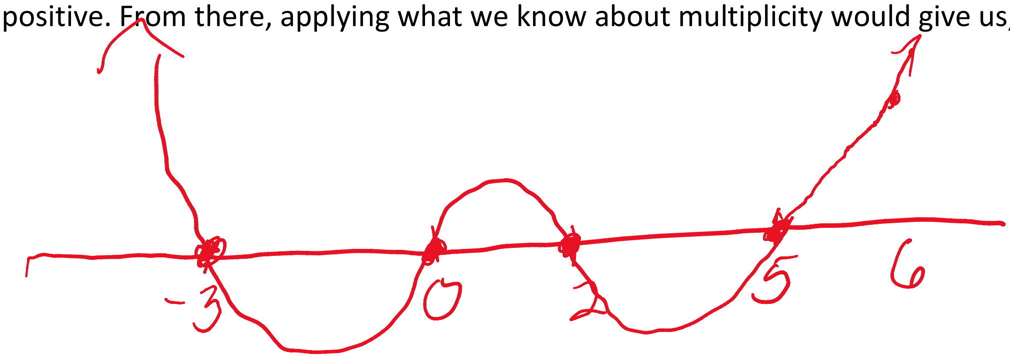
Are the graphs consistent with your solutions to #2 and #3, and why?

Question 4 is the geometric approach to the problems by graphing $y = |x-1|$ *and* $y = 4$. *They would be equal at points of intersection(Question 1). The absolute value would be less than 4 in the interval where the absolute value graph is below the horizontal line (Question 2). The absolute value would be greater in the intervals where the absolute value graph is above the horizontal line.*

Task 2: Polynomial Inequality **Name: ____________________**

1. Let's consider a rough sketch of the function $\boldsymbol{f(x) = x(x-2)(x+3)(x-5)}$

   We know the zeros of this function are $x = 0, 2, -3, 5$. We know that all of these zeros have multiplicity one (because each happens only once). We also know that functions cross the axis at zeros of odd multiplicity (PLEASE REVIEW FROM LESSON 7.1 if confused). So we are able to ***roughly*** sketch what is going on with this function by choosing one value of x that is not a zero. For instance, if we choose x=6, we can look at the factors x, x-2, x+3, and x-5. Plugging in x=6, we discover all factors are positive. Multiplying 4 positive factors would yield a positive answer, so we know when x is 6, the function is positive. From there, applying what we know about multiplicity would give us,



   We know if we expanded f(x) we would get a 4^{th} degree polynomial. By looking at the end behavior, we know that our graph makes sense. It is fine to insert a y-axis, but again for what we will be looking at, it is not necessary.

2. Represent the solution to $\boldsymbol{x(x-2)(x+3)(x-5) \leq 0}$ using:

   We simply need to look at the graph and see where it is on or below the x-axis

   Interval notation: $[-3,0] \cup [2,5]$

   An inequality(s): $-3 \leq x \leq 0,\ 2 \leq x \leq 5$

3. Represent the solution to $\boldsymbol{x(x-2)(x+3)(x-5)} > 0$ using:

Again, we simply need to look at the graph and see where it is above the x-axis

Interval notation: $(-\infty,-3) \cup (0,2) \cup (5,\infty)$

An inequality(s): $x < -3, 0 < x < 2, x > 5$

4. Represent the solution to $\boldsymbol{9x \geq x^3}$ using: **(Do NOT use a calculator to graph!)**

Put the inequality in standard form: $x^3 - 9x \leq 0$

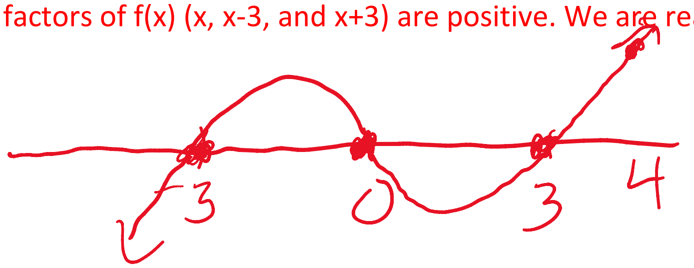
We will need to consider the above as a function: $f(x) = x^3 - 9x$.

Now, we need it in factored form: $f(x) = x(x-3)(x+3)$.

Next find the zeros: x=0, 3, -3. All zeros have multiplicity 1 (odd multiplicity)

Graph the zeros and pick a point that is not a zero (I choose x=4)

When x=4, all factors of f(x) (x, x-3, and x+3) are positive. We are ready to sketch



Now look for where the function is less than or equal to zero

Interval notation: $(-\infty,-3] \cup [0,3]$

An inequality(s): $x \leq -3, 0 \leq x \leq 3$

THERE ARE 5 PRACTICE PROBLEMS ON THE NEXT PAGE. DO THESE FIRST AND CHECK WITH THE PROVIDED ANSWER SHEET. THEN THE HOMEWORK IS IMMEDIATELY AFTER (ANSWERS PROVIDED)

Task 3: Practice Name: ______________________

Solve each inequality. Give your answer in interval notation.

1. $|\mathbf{2x-3}|-\mathbf{5}>0$

2. $|\mathbf{3x-5}|\le -\mathbf{2}$

3. $\mathbf{2x^2+2x}\ge \mathbf{112}$

4. $\mathbf{x^4-3x^3}<4\mathbf{x^2}$

5. $\mathbf{x-2}>|\mathbf{x^2-4}|$

HOMEWORK

1. $|x+4| \geq 5$ $(-\infty,-9] \cup [1,\infty)$
2. $|x-3| < 2$ $(1,5)$
3. $|4-3x| - 2 < 4$ $(-\frac{2}{3},\frac{10}{3})$
4. $\left|\frac{x+2}{3}\right| \geq 3$ $(-\infty,-11] \cup [7,\infty)$
5. $(x+1)(x-3)^2 > 0$ $(-1,3) \cup (3,\infty)$
6. $(x+1)(x^2-3x+2) < 0$ $(-\infty,-1) \cup (1,2)$
7. $2x^3 - 3x^2 - 11x + 6 \geq 0$ $\left[-2,\frac{1}{2}\right] \cup [3,\infty)$
8. $x^3 - x^2 - 2x \geq 0$ $[-1,0] \cup [2,\infty)$
9. $2x^3 - 5x^2 - x + 6 > 0$ $(-1,\frac{3}{2}) \cup (2,\infty)$