Pre Calculus
Lesson 7.1 Power Functions and Modeling

Power Functions: $y = kx^a$

Examples: Analyze the functions

	1. $f(x) = x^3$	2. $f(x) = x^3 - x$
Domain	$(-\infty, \infty)$	$(-\infty, \infty)$
Range	**Reals**	**Reals**
Inc/Dec	**Increase throughout** **Decrease: never**	**Inc** $(-\infty, -1) \cup (1, \infty)$ **Dec** $(-1, 1)$
Continuity	**Continuous**	**Continuous**
Boundedness	**Not bounded**	**Not bounded**
Symmetry	**Origin**	**Origin**
Extrema	**None**	**rel max 0.38, rel min -0.38**
Asymptotes	**None**	**None**
End Behavior	$\lim_{x\to\infty} f(x) = \infty$ $\lim_{x\to-\infty} f(x) = -\infty$	$\lim_{x\to\infty} f(x) = \infty$ $\lim_{x\to-\infty} f(x) = -\infty$

YOU MAY GRAPH ON YOUR CALCULATOR FOR #2. Sketch below

Polynomial Function: $f(x) = a_n x^n + a_{n-1}x^{n-1} a_{n-2}x^{n-2} + ... + a_2 x^2 + a_1 x + a_0$**, where n is a nonnegative integer and** $a_0, a_1, a_2, ... a_{n-1}, a_n$ **are reals where** $a_n \neq 0$**.**

So as we look at the two analyses, we see both similarities and differences.

WHAT IS THE DIFFERENCE BETWEEN A POWER AND A POLYNOMIAL FUNCTION? A power function is one term raised to a power and a polynomial function is the sum/difference of multiple terms raised to powers. In the above examples, #1 is a power function and #2 is a polynomial function.
***Keep in mind, $f(x) = x^3 - 5$ is still considered a power function as there is still only one term raised to a power.**

It is a good idea to review the idea of degree at this point.
<u>Degree of a power function</u>: Sum of the exponents on the variables

<u>Degree of a polynomial function</u>: the highest degreed term gives the degree of the polynomial.

Ex) Sketch each function (Calculator active). Locate the extrema and the zeros.

1. $f(x) = x^3 + x$

Extrema:* *none

Zeros:* *x=0

2. $f(x) = x^3 - x$

Extrema:* *rel max: 1, rel min: -1

Zeros:* *x=-1, 0, 1

Notice, both are degree 3 polynomials. Using the above example, we can draw a conclusion about the local (relative) extrema and the number of zeros of a function.

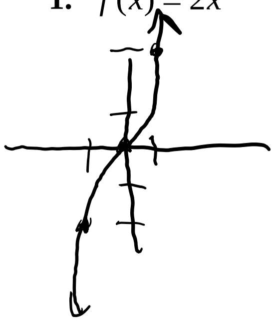
<u>Local Extrema and Zeros of Polynomial Functions:</u> *A polynomial of degree n has <u>at most</u> (n-1)local extrema and <u>at most</u> n zeros.*

The examples should explain why we have to say at most. As you can see both were degree 3, yet #1 had 1 zero and #3 had 3 zeros. How can we account for this?
Zeros indicate where a function crosses the x-axis and while every polynomial equation of degree 3 should have 3 solutions, some of those may be complex solutions. Zeros are real numbers because they indicate where a function crosses the x-axis.

<u>**End Behavior of Polynomial Functions**</u>

Examples) Determine the end behavior for the following functions (sketch a graph)

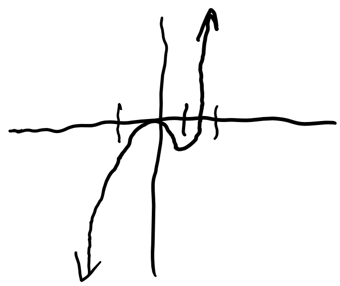
1. $f(x) = 2x^3$



$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

2. $f(x) = 2x^3 - 2x^2 - x$ **(calculator active)**



$\lim_{x\to\infty} f(x) = \infty$

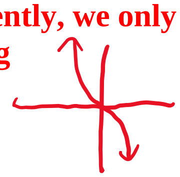
$\lim_{x\to-\infty} f(x) = -\infty$

Notice a third degree power function has the same end behavior as a third degree polynomial function. The polynomial function offers the possibility of some turns in it; however, that does not impact what happens at the ends of the function, so <u>end</u> behavior does not change.

So the value of knowing this, I will not need a calculator to determine the end behavior of a polynomial function. It will behave the same as its corresponding power function (which we already know from our toolkit functions!)

Without a calculator determine the end behavior of $f(x) = -5x^7 + 2x^4 - 3x + 2$

So, we are not concerned with the turns within the polynomial, we on are concerned with the ends of the function. Consequently, we only need to think of $-5x^7$. In our heads, very roughly, we should be seeing



and that is enough to give the end behavior of the polynomial.

Final answer

$\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to-\infty} f(x) = \infty$

Zeros of Polynomial Functions

Ex 1) Find the zeros of $f(x) = x^3 - x^2 - 12x$

$$x(x^2 - x - 12) = 0$$
$$x(x-4)(x+3) = 0$$
$$x = 0, x = 4, x = -3$$

Ex 2) Find the zeros of $f(x) = x^4 - 6x^3 + 9x^2$

$$x^4 - 6x^3 + 9x^2 = 0$$
$$x^2(x^2 - 6x + 9) = 0$$
$$x^2(x-3)(x-3) = 0$$
$$x = 0, x = 3$$

0 and 3 are what you may have referred to, in the past, as double roots. We don't write x=3, x=3 but we know it happens twice. As we look at polynomials of higher degree, we need to change the terminology for times when a root occurs more than once (as opposed to saying "quintuple root" for instance.)

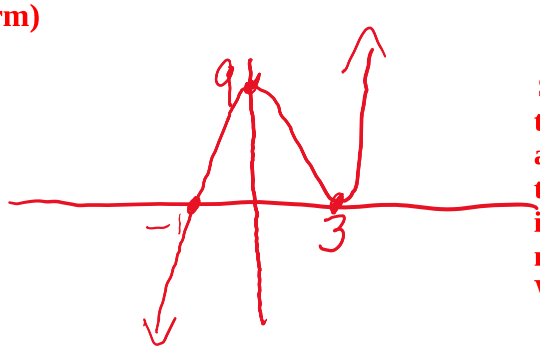
Multiplicity: **Used when a zero occurs more than once.**

Ex) $f(x) = (x-3)^2$ **x=3 is a zero of multiplicity 2**

Ex) Sketch a graph of $g(x) = (x-3)^2(x+1)$ **Identify the zeros.**

In factored form, the zeros are easily identified $x = 3\ (mult\ 2), x = -1\ (mult\ 1)$**. We do not need to indicate mult 1. It would be understood if we simply wrote x=-1.**

Prior to graphing we also want to think that in expanded form (multiplied out), this is a third degree polynomial, so we should know what the ends look like. For this example, use your calculator to graph this function (you may leave it in factored form)



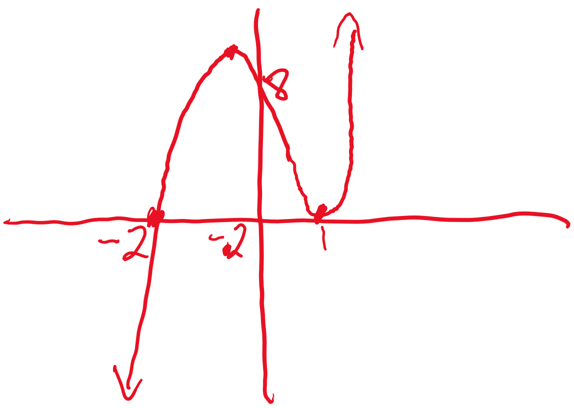
So, if we know a 3rd degree polynomial the ends are not a surprise. But, look at what happens as the function hits the zeros. Going from left to right, it hits x=-1 first. This zero has odd multiplicity since it happens only once When a function hits a zero of odd multiplicity, the function will cross the x-axis at that point. Next is the y-intercept (let x=0). Then the function travels to the next zero, x=3 which has even multiplicity (2). When a function hits a zero of even multiplicity, it will touch the zero and stay on the same side of the x-axis

So, in summary:

<u>Zeros of odd multiplicity</u>- cross the x-axis and the value of f changes sign at that point.

<u>Zeros of even multiplicity</u>—do not cross the x-axis and the value of f does not change sign at that point.

Ex) Draw a rough sketch a graph of $f(x) = (x+2)^3(x-1)^2$ by hand if there is a relative max of approximately 8.4 when x is approximately -0.2.



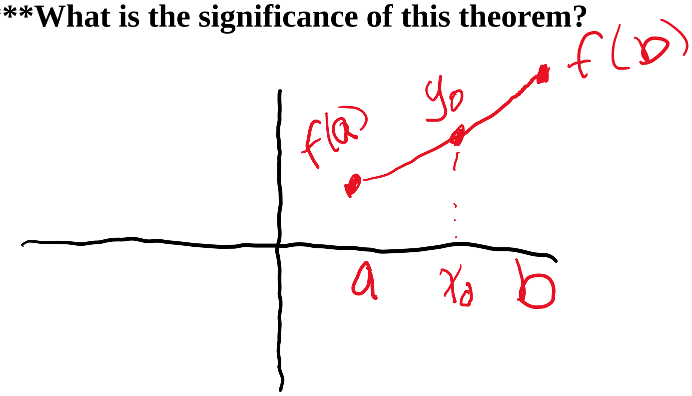
HW pp 202-204: 1-5 odd; 7(just give end behavior and zeros), 9,11, 17-23 odd (end behavior only, no calc), 25-39 odd, 53

BELOW IS OPTIONAL WORK ON THE INTERMEDIATE VALUE THEOREM. PURSUE ONLY IF YOU HAVE TIME

OPTIONAL

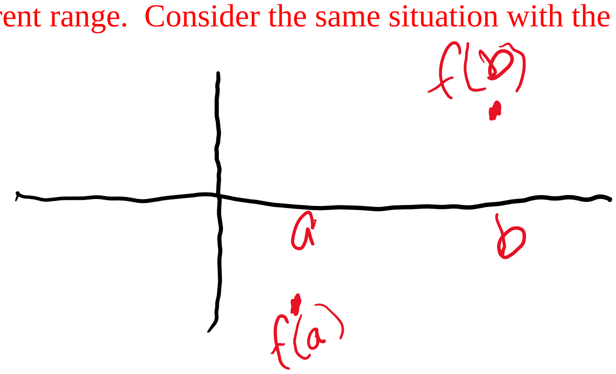
<u>Intermediate Value Theorem:</u> Suppose f is a continuous real function on an interval, I, and a and b are on I. Then for every $\Re y_0$ between f(a) and f(b), there is at least one $\Re x_0$ between a and b such that $f(x_0) = y_0$.

*****What is the significance of this theorem?**



This theorem really seems to establish "betweenness" for continuous functions. CONTINUITY IS EXTREMELY IMPORTANT HERE! For example if I am looking at a partial range like $2 \le y \le 5$ on an interval of (2, 6) and the function is continuous in that space then I know that ALL real numbers between 2 and 5 exist in the range. So, for instance I could safely say that a y-value of 3.2 would have a corresponding x-value somewhere between 2 and 6.

And so the big question: SO WHAT? WHY DO I CARE? Well, let's consider a different range. Consider the same situation with the following drawing.



Now we care! If the function is continuous in the interval (a,b), then I can draw a smooth unbroken curve between the points and determine there is a zero in the interval (a,b). If it is not continuous, then I cannot make the assumption.

Example: If the data in the table are points on a continuous function, determine an interval of length 1 such that f(x)= -1

X	-2	-1	0	1	2
Y	-2.35	-1.78	-1.5	-0.76	1

With the guarantee that the function is continuous we simply need to look at the y-values. We see that our graph would have to have hit a y-value of -1 in the interval (0,1)

Your calculator uses the intermediate value theorem to find zeros, max points and min points. Think about this, in all of those operations, we are asked for a left bound and a right bound to establish a point somewhere between these bounds. This is Intermediate Value Theorem!