Remote Lesson (Optional)
Lesson 6.6 Surface Area and volume

Ex) A container manufacturer wants a cylindrical metal can that will hold one quart of paint with a little extra room for air space. One liquid quart occupies a volume of 57.749 cubic inches, so he decides to design the can with a volume of 58 cubic inches. Of course he wants to keep cost at a minimum. How should this be done? Give the dimensions of the can.

Purpose: Minimize the amount of metal needed (surface area) to keep production cost down.

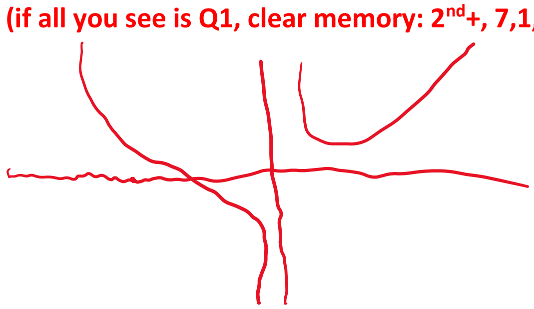
Surface Area: Area of all surfaces added together. Imagine taking the can apart. The top and the bottom would be circles ($Area = \pi r^2$). The "body" of the can could be unwrapped to form a rectangle ($Area = b \times h$). The base of the body, if wrapped back up would be the circumference of a circle ($C = 2\pi r$).

So the surface area of our can would be: $SA = 2\pi r^2 + 2\pi rh$.
The issue here is we have one equation with 2 variables, so we cannot yet minimize.

The other quantity given in this problem is a volume of 58 cubic inches.
Volume is found by taking the area of the base, in this case a circle, times the height
$V = \pi r^2 h$
$58 = \pi r^2 h$
$\frac{58}{(\pi r^2)} = h$ **Note the parentheses around the denominator**

We have just created a substitution for *h* in our surface area equation

$SA = 2\pi r^2 + 2\pi r \frac{58}{(\pi r^2)}$
Or in simplified form: $SA = 2\pi r^2 + \frac{116}{r}$
This is now an equation with one variable, which allows us to look geometrically (on a graph) at our surface area function.
On our calculator, $y = 2\pi x^2 + \frac{116}{x}$ **.**
ZOOM 0:fit (if all you see is Q1, clear memory: 2nd+, 7,1,2 and regraph equation)



In the drawing, we need to consider the context of the problem, so negative values of x and y make no sense, but we are able to look at the minimum point in Q1. *2nd TRACE 3:minimum* and we locate a min at $x \approx 2.098, y \approx 82.947$ *(as it appears on calculator screen but remember the min is the y-value)*

We now need to reapply the context of the problem, remembering the y-value would represent surface area and the x-value would represent the radius of the can.

To find the remaining dimension, height, we will take our radius value and substitute it in the easiest place above ($h = \frac{58}{\pi r^2}$) . So $h \approx 4.196\ in$

Conclusion (final answer): Design the can with a radius of approximately 2.098 in, a height of approximately 4.196 in, to yield a minimum surface area of 82.947 cubic inches

Ex) A rancher is planning to fence in a rectangular area of 2000 square meters for his cattle to graze. He wants to use the least amount of fencing. How can this be done?

Purpose: Use the least amount of fencing (perimeter) in order to minimize cost.

In this problem we are looking at a rectangle and we know $P = 2L + 2W$
Again, we have 2 variables in the equation so we will use the other quantity they gave us.
Area is given and we know area of a rectangle: $A = LW$
So $2000 = LW$. We can solve for either L or W.
$L = \frac{2000}{W}$. We will now plug this substitution into our perimeter formula
$P = 2\left(\frac{2000}{W}\right) + 2W$, or in simplified form,
$P = \frac{4000}{W} + 2W$. We will look at the graph on the calculator
$y = \frac{4000}{x} + 2x$. ZOOM:fit

What we see is a graph that does not appear to have a minimum. We will have to go to the WINDOW and make adjustments. Again, in the context of the problem, negative values of width and perimeter make no sense, so we will reset these to 0. In order to find a minimum, we need to see a function decrease and then increase, so we will have to expand the x-values to some arbitrary amount (I always choose 100 to start for xmax). We also have to "dial in" the ymax value (right now is approx. 18,800). Again arbitrary (I always choose 500 and adjust from there if needed). With these adjustments, if we now press GRAPH, we will see a minimum taking shape. 2nd TRACE: 3min. BE CAREFUL—when using this feature you are asked for a left bound and a right bound. Be sure you are to the left of the min, press ENTER, then as you look for the right bound, watch the y-values! You need to see them go down and back up again, THEN you have found a right bound.

Minimum: $x \approx 44.721, y \approx 178.885$
Reapply the context: width is 44.721, perimeter is 178.885. GO BACK AND FIND LENGTH.
Length is 44.721
Conclusion*: Design the fence with a length and width of 44.721 meters for a minimum perimeter of 178.885 meters*

Ex) A square side of x inches is cut out of each corner of an 8 in. X 15 in. piece of cardboard and folded up to make a box.

a) **Write a formula for V as a function of x**
b) **Consider the domain**
c) **Find the maximum volume of the box**

In this problem, imagine using a pair of scissors to cut the x by x squares out of each corner and watching the cardboard squares fall to the floor. If you now fold along the dotted lines, you could create a box with no top. The length of the box would now be 15-2x, the width of the box would be 8-2x, and the height of the box is x.

a) **Volume as a function of x: $V = (15 - 2x)(8 - 2x)x$. Note- there is NO NEED to multiply this out. It is an equation with one variable, so we will take this, as is, to the calculator.**

$y = (15 - 2x)(8 - 2x)x$. **ZOOM fit and look for a max**

Our graph has proved less than helpful again, so we will consider the domain

b) **If we consider our volume equation above in its nice, factored form and combine that with the context of our problem (x represents height), then we realize that x values greater than 4 will yield negative width (remember width is 8-2x). So as we adjust the window, we know an x max value should be set at 4. We will adjust the rest of the window as it was prescribed in the previous problem)**
c) **Find the maximum 2nd TRACE 4:maximum** ***Max volume is approximately 90.739in^3***

The intent of this lesson is not only minimizing and maximizing but to think through what you know and adjust calculator windows to suit your needs, taking you to the places on the graph you want to see. ZOOM Standard and ZOOM fit are great tools, but as we learned here, they will not know what application we are applying it to. It is worth rereading the notes in this lesson to help better ground you in your calculator skills.

Pre Calculus
Unit 6 Applications Lesson

1. The manufacturer discussed in the first example from the notes is also commissioned to make cylindrical cans with volume $58\ in^3$ and no tops because these will be closed with plastic lids. Once again the manufacturer is interested in finding the dimensions which require the least amount of metal for each can. Determine the approximate values for the optimum dimensions. $(r \approx 2.64\ in, h \approx 2.65\ in, SA \approx 65.8 in^3)$

2. Suppose that a cylindrical can is to hold a liter of oil. Recall that1 $liter = 1000\ cm^3$.
   a.) Give a formula for the surface area of this can in terms of its radius
   $(SA = 2\pi r^2 + \frac{2000}{r})$
   b.) Approximately what radius and height (cm) would use the least amount of material to make the can? $(r \approx 5.41 cm, h \approx 10.9 cm, SA \approx 553 cm^3$

3. A box is to be constructed with a square base and no top. It must hold a volume of 10 cubic meters but be made with a minimum of materials. Let *s* be the length of a side and let *h* be the height of the box.
   a.) Find a formula for h in terms of s. $(h = \frac{10}{s^2})$
   b.) Find a formula for the surface area of the box as a function of s. $(SA = s^2 + \frac{40}{s})$
   c.) Estimate the value of s which minimizes the surface area. $(s \approx 2.7\ m)$
   d.) What are the approximate dimensions and surface area of the box with minimal surface area? $(2.7m \times 2.7m \times 1.4m\ for\ a\ surface\ area\ of\ \approx 221 m^2$