Remote lesson 6.2

Increasing and Decreasing Intervals, Symmetry, Range, Boundedness, Extrema, Limits

Determine the intervals over which the function is increasing and decreasing. ***Use open intervals unless the function has an endpoint. Remember intervals are x-values!***

Final answer: $Increasing: \left[-8,-5\right)∪\left(1,\infty \right)$

$$Decreasing: (-5, 1)$$



**Final Answer:** $Increasing (-\infty , -1)∪(-1, 0)$

 $Decreasing \left(0,1\right)∪\left(1,\infty \right)$

**\*Remember you are looking at the x-values over which the function goes up and down as you read from left to right. Stay focused on the x-axis as you answer!**

**Symmetry**

With respect to the y-axis: **Algebraic**: f(-x) = f(x)

 **Geometric**: Looking at a graph, these are

 Even Functions (Remember identities) opposite x values yield equal y-values

With respect to the origin: **Algebraic**: f(-x) = -f(x)

 **Geometric**: Looking at a graph, these are

 Odd Functions—opposite x values yield opposite y-values.

With respect to the x-axis: If (x,y) is on the graph (x, -y) is on the graph **Not Functions**

***Please note: If for example an even function moves 4 to the right, we would use the term “line symmetry at x=4” as opposed to y-axis symmetry***

***Similarly, we would call it point symmetry if an odd power function shifted as opposed to origin symmetry***

Ex) Determine the symmetry algebraically

1) 

 Start by finding f(-x) – take all x values and replace with -x

 $f\left(-x\right)= (-x)^{2}+4$ Note the importance of parentheses when replacing x with -x

 $f\left(-x\right)=x^{2}+4$ Simplify the expression.

 Note that the right hand side of f(x) is the same as the right hand side of f(-x)

 None of the signs changed when comparing the answers

 Therefore f(-x) = f(x)

 Final answer: this function has y-axis symmetry

2) $f\left(x\right)=-x^{3}-x$

 $f\left(-x\right)=-(-x)^{3}-(-x)$

 $f\left(-x\right)=x^{3}+x$ notice ALL signs changed compared to f(x)

 This function has origin symmetry

3) 

 $f\left(-x\right)=(-x)^{4}-2\left(-x\right)+4$

 $f\left(-x\right)=x^{4}+2x+4$

 Some signs changed, some didn’t

 Therefore this function has no symmetry

4) 

 $f\left(-x\right)= \frac{(-x)^{3}}{9-(-x)^{2}}$

 $f\left(-x\right)=\frac{-x^{3}}{9-x^{2}}$

 $f\left(-x\right)=-\frac{x^{3}}{9-x^{2}}$

 The last fraction is the opposite of the original.

 Therefore this function has origin symmetry

**Range: The y-values generated by evaluating the function throughout the domain.**

 **Geometrically: Graph and observe the y-values of the function**

 **Algebraically: Find the inverse of the function and find its domain**

$The range of f\left(x\right)=the domain of f^{-1}(x)$

Ex) Find the range of the function: ***For now, you can graph on your calculator if needed***

1. f(x) = x2 + 3 This is a parabola whose vertex is moved up 3.

 Final answer: y > 3 (book will say $[3, \infty ))$

1. g(x) =  This is a square root function translated 4 units left

 Final answer: y > 0

1. h(s) =  This function is vertically stretched by $\frac{\sqrt{10}}{3}$

 Final answer All Reals

**Bounded**

Def. A function is **bounded below** if there exists some number, b, less than or equal to every number in the range of f. b is the **lower bound.**

Def. A function is **bounded above** if there exists some number, b, greater than or equal to every number in the range of f. b is the **upper bound.**

\*\*A function is **bounded** if it is bounded above and below.

**Extrema**

A **relative (local) maximum of f** is a maximum value within an interval of a function. The absolute max of the function may also be a relative max, as long as it can be contained in an interval. *To be a relative max, the function must increase to the point and decrease after.*

Please give only the y-value here (so as not to confuse it with an interval)

A **relative (local) minimum of f** is a minimum value within an interval of a function. The absolute min may also be the relative min. *To be a relative min, the function must decrease to the point and increase after.*

Again, please give y- value

**Extrema includes Max, min, relative max and relative min**

Ex) Given f(x) = 2x4-5x2+2x, find all extrema (Use 2nd trace on TI)

 local min: -5.453 and -1 min: -5.453

 local max: 0.203 max: none

**Limits***: You have done these as* $x\rightarrow \infty , y\rightarrow and x\rightarrow -\infty , y\rightarrow $ *we are simply changing the notation*

Ex) Lets look at even power functions,

 f(x)=x2



As x-increases, what happens to y?

 limf(x)= $\infty $ or limx2= $\infty $

x-> ∞ x->∞

As x decreases?

 limf(x)=$\infty $

 x->-∞

*Think of f(x) and y as the same thing. The first statement is read*

 *“the limit of f(x) as x approaches infinity”*

Ex) Odd power

 f(x)=x3

 limf(x)=$\infty $

 x->∞

 limf(x)=$-\infty $

 x->-∞

**End Behavior: when asked for end behavior you will need to write and find both** $\lim\_{x\to \infty }f(x) and \lim\_{x\to -\infty }f(x)$

**Analyzing a Function:** when you are asked to analyze a function, you will need to answer the following

1. Domain: Use Interval Notation
2. Range: Use Inequalities
3. Increasing and Decreasing Intervals: Answer inc: dec:
4. Continuity: Answer “*continuous” or something like “essential discontinuity at x= “*
5. Symmetry: Answer “y-axis”, “origin”, “line at x=”, or “point at (x,y)”
6. Boundedness Answer “above”, “below”, “bounded” or “not bounded”
7. Extrema Address max, min, rel max, rel min (y-values)
8. Asymptotes Both vertical and horizontal
9. End Behavior $\lim\_{x\to \infty }f(x) and \lim\_{x\to -\infty }f(x)$
10. Graph: If not given to you, graph using a minimum of 3 anchors

On the next page a graph is given, the analysis would look like this

1. Domain: $(-\infty ,\infty )$
2. Range: y > -75
3. Increase: $(-2,1)∪(4,\infty )$ Decrease $(-\infty ,-2)∪(1,4)$
4. Continuous
5. No Symmetry
6. Bounded below
7. No max, min: -75, rel max: $≈35$, rel min: $≈-35 and-75$
8. No asymptotes
9. $\lim\_{x\to \infty }f(x)=\infty and \lim\_{x\to -\infty }f\left(x\right)=\infty $

**Example Analyze the graph**



HOMEWORK: P 98 (1-53 Every other odd) Use a calculator when needed to look at graphs