Remote lesson 6.2
Increasing and Decreasing Intervals, Symmetry, Range, Boundedness, Extrema, Limits

Determine the intervals over which the function is increasing and decreasing. ***Use open intervals unless the function has an endpoint. Remember intervals are x-values!***

1.

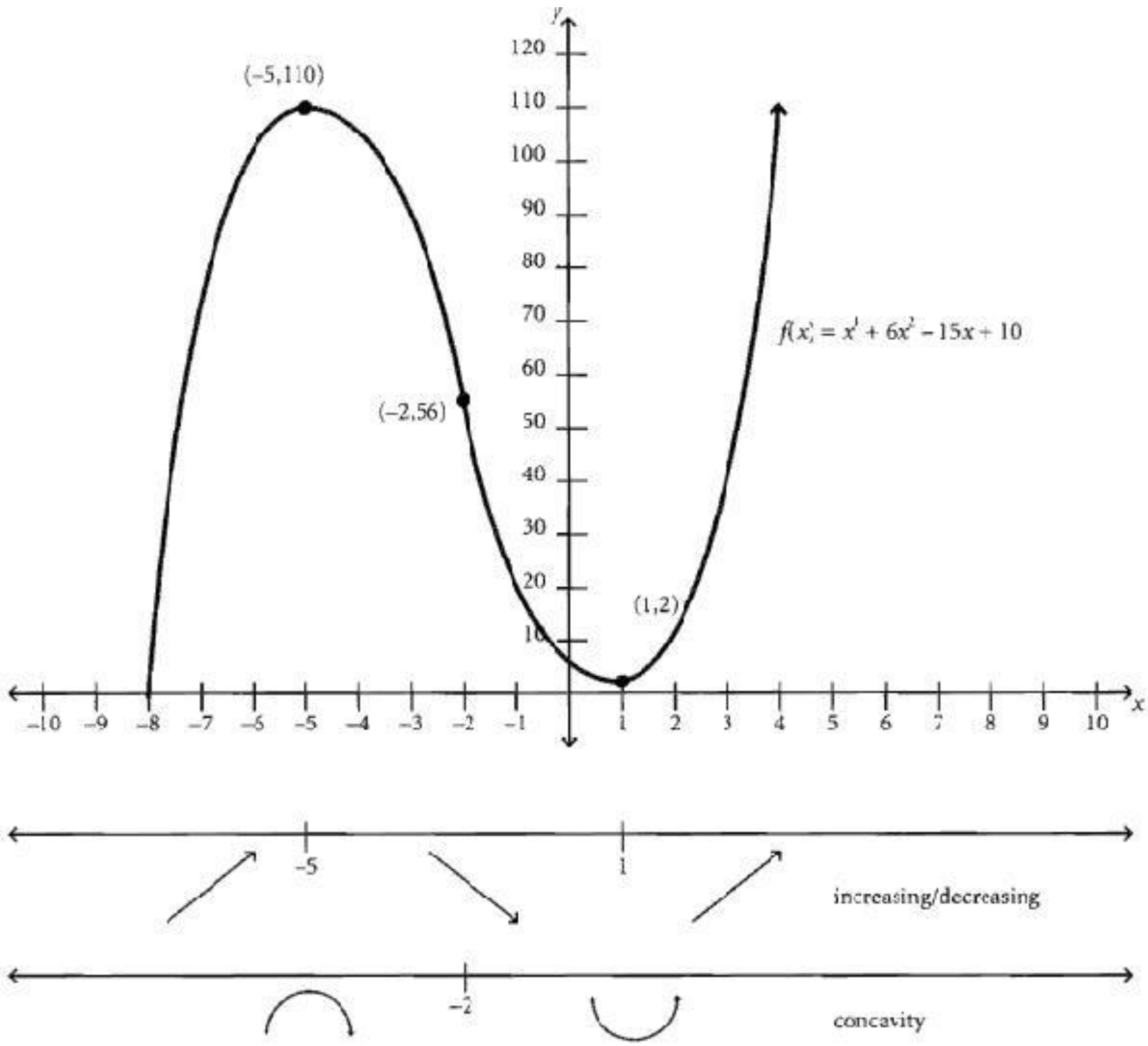


Figure 14.10

Final answer: $Increasing$: $[-8,-5) \cup (1,\infty)$

$Decreasing$: $(-5,1)$

2.

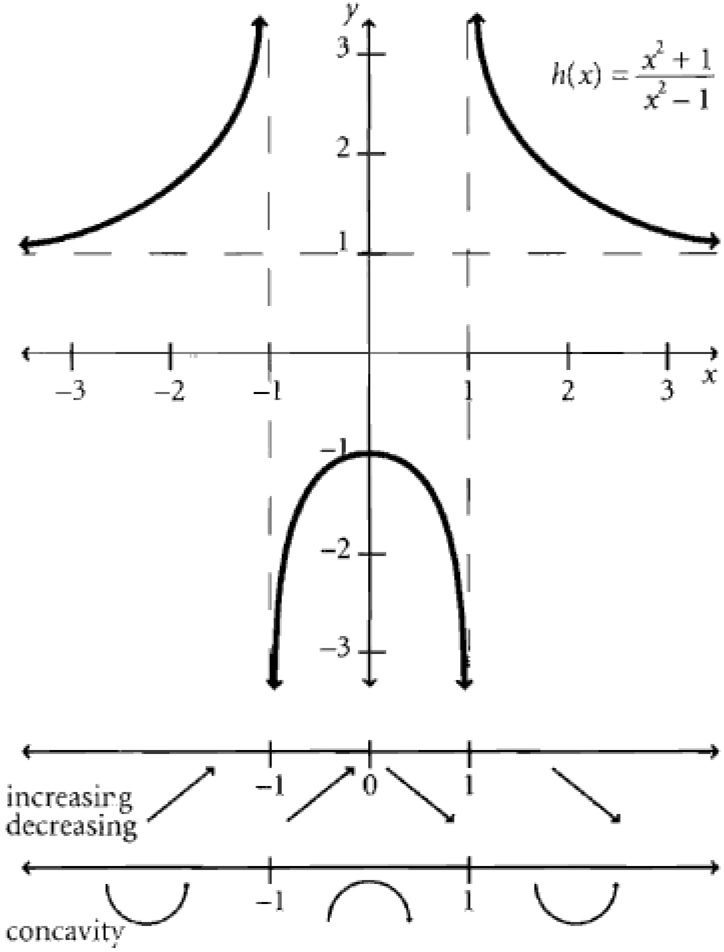


Figure 14.20

Final Answer: $Increasing\ (-\infty,-1) \cup (-1,0)$

$Decreasing\ (0,1) \cup (1,\infty)$

***Remember you are looking at the x-values over which the function goes up and down as you read from left to right. Stay focused on the x-axis as you answer!**

Symmetry

With respect to the y-axis: **Algebraic**: f(-x) = f(x)
Geometric: Looking at a graph, these are
Even Functions (Remember identities) opposite x values yield equal y-values

With respect to the origin: **Algebraic**: f(-x) = -f(x)
Geometric: Looking at a graph, these are
Odd Functions—opposite x values yield opposite y-values.

With respect to the x-axis: If (x,y) is on the graph (x, -y) is on the graph **Not Functions**

Ex) Determine the symmetry algebraically

1) $f(x) = x^2 + 4$
Start by finding f(-x) – take all x values and replace with -x
$f(-x) = (-x)^2 + 4$ Note the importance of parentheses when replacing x with -x
$f(-x) = x^2 + 4$ Simplify the expression.
Note that the right hand side of f(x) is the same as the right hand side of f(-x)
None of the signs changed when comparing the answers
Therefore f(-x) = f(x)
Final answer: this function has y-axis symmetry

2) $f(x) = -x^3 - x$
$f(-x) = -(-x)^3 - (-x)$
$f(-x) = x^3 + x$ notice ALL signs changed compared to f(x)
This function has origin symmetry

3) $f(x) = x^4 - 2x + 4$

$f(-x) = (-x)^4 - 2(-x) + 4$

$f(-x) = x^4 + 2x + 4$

Some signs changed, some didn't

Therefore this function has no symmetry

3) $f(x) = \dfrac{x^3}{9 - x^2}$

$f(-x) = \frac{(-x)^3}{9-(-x)^2}$

$f(-x) = \frac{-x^3}{9-x^2}$

$f(-x) = -\frac{x^3}{9-x^2}$

The last fraction is the opposite of the original.

Therefore this function has origin symmetry

<u>Range</u>: The y-values generated by evaluating the function throughout the domain.

Geometrically: Graph and observe the y-values of the function

Algebraically: Find the inverse of the function and find its domain

$\boldsymbol{The\ range\ of\ f(x) = the\ domain\ of\ f^{-1}(x)}$

Ex) Find the range of the function: ***For now, you can graph on your calculator if needed***

1.) f(x) = x^2 + 3 This is a parabola whose vertex is moved up 3.

Final answer: y $\geq$ 3 (book will say $[3, \infty)$)

2.) g(x) = $\sqrt{x+4}$ This is a square root function translated 4 units left

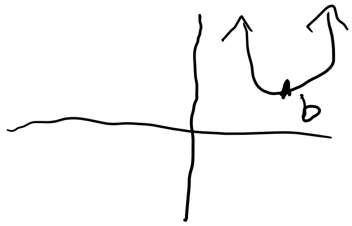
Final answer: y $\geq$ 0

3.) h(s) = $\dfrac{\sqrt{10}}{3}s^3$ This function is vertically stretched by $\frac{\sqrt{10}}{3}$

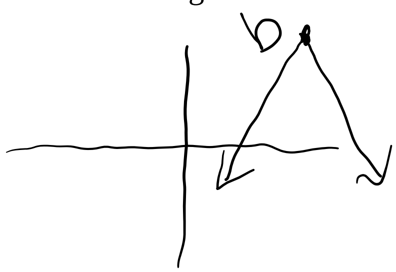
Final answer All Reals

Bounded

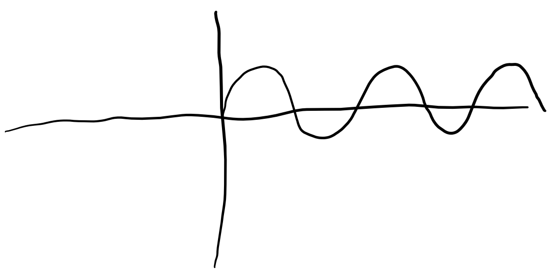
Def. A function is **bounded below** if there exists some number, b, less than or equal to every number in the range of f. b is the **lower bound.**



Def. A function is **bounded above** if there exists some number, b, greater than or equal to every number in the range of f. b is the **upper bound.**



A function is **bounded if it is bounded above and below.



Extrema

A **relative (local) maximum of f** is a maximum value within an interval of a function. The absolute max of the function may also be a relative max, as long as it can be contained in an interval. *To be a relative max, the function must increase to the point and decrease after.* Please give only the y-value here (so as not to confuse it with an interval)

A **relative (local) minimum of f** is a minimum value within an interval of a function. The absolute min may also be the relative min. *To be a relative min, the function must decrease to the point and increase after.*
Again, please give y- value

Extrema includes Max, min, relative max and relative min

Ex) Given $f(x) = 2x^4-5x^2+2x$, find all extrema (Use 2nd trace on TI)

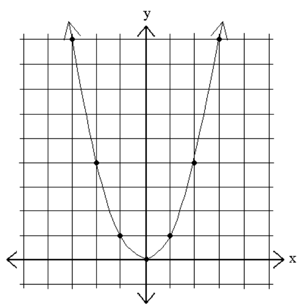
local min: -5.453 and -1 min: -5.453

local max: 0.203 max: none

Limits: *You have done these as* $x \to \infty, y \to$ *and* $x \to -\infty, y \to$ *we are simply changing the notation*

Ex) Lets look at even power functions,

$f(x)=x^2$



As x-increases, what happens to y?

$\lim\limits_{x \to \infty} f(x) = \infty$ or $\lim\limits_{x \to \infty} x^2 = \infty$

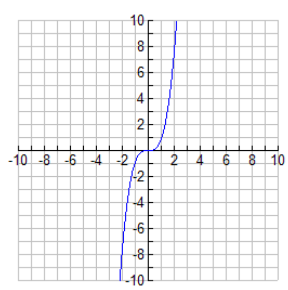
As x decreases?

$\lim\limits_{x \to -\infty} f(x) = \infty$

Think of f(x) and y as the same thing. The first statement is read "the limit of f(x) as x approaches infinity"

Ex) Odd power

$f(x)=x^3$



$\lim\limits_{x \to \infty} f(x) = \infty$

$\lim\limits_{x \to -\infty} f(x) = -\infty$

End Behavior: when asked for end behavior you will need to write and find both $\lim\limits_{x\to\infty} f(x)$ ***and*** $\lim\limits_{x\to-\infty} f(x)$

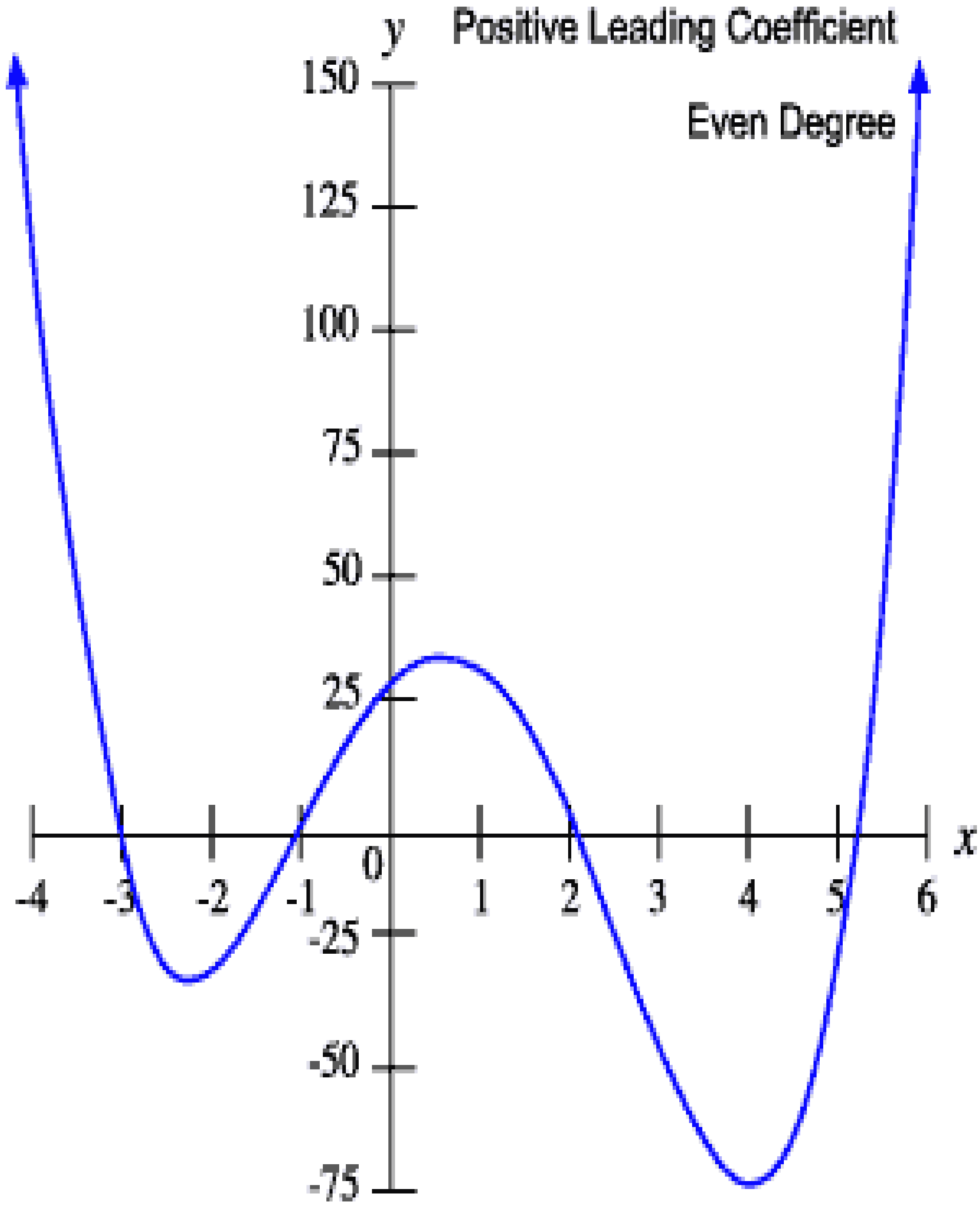
Analyzing a Function: when you are asked to analyze a function, you will need to answer the following

1. Domain: Use Interval Notation
2. Range: Use Inequalities
3. Increasing and Decreasing Intervals: Answer inc: dec:
4. Continuity: Answer "*continuous" or something like "essential discontinuity at x=* "
5. Symmetry: Answer "y-axis", "origin", "line at x=", or "point at (x,y)"
6. Boundedness Answer "above", "below", "bounded" or "not bounded"
7. Extrema Address max, min, rel max, rel min (y-values)
8. Asymptotes Both vertical and horizontal
9. End Behavior $\lim_{x\to\infty} f(x)$ *and* $\lim_{x\to-\infty} f(x)$
10. Graph: If not given to you, graph using a minimum of 3 anchors

On the next page a graph is given, the analysis would look like this

1. Domain: $(-\infty, \infty)$
2. Range: y $\geq$ -75
3. Increase: $(-2,1) \cup (4, \infty)$ Decrease $(-\infty, -2) \cup (1,4)$
4. Continuous
5. No Symmetry
6. Bounded below
7. No max, min: -75, rel max: ≈ 35, rel min: ≈ -35 *and* -75
8. No asymptotes
9. $\lim_{x\to\infty} f(x) = \infty$ *and* $\lim_{x\to-\infty} f(x) = \infty$

Example Analyze the graph



HOMEWORK: P 98 (1-53 Every other odd) Use a calculator when needed to look at graphs