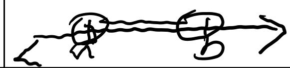
Lesson 6.1 Domain

In order to discuss domain, we need a working understanding of intervals.

<u>Interval Notation</u>

Name	Interval notation	Inequality	Graph
Open	(a,b)	a< x< b	
Half open	[a,b) Or (a,b]	a ≤ x < b a < x ≤ b	
Closed	[a, b]	a ≤ x ≤ b	
Open Infinite	(a, ∞) Or $(-\infty, a)$	x > a x < a	
Closed Infinite	$[a, \infty)$ Or $[-\infty, a)$	x ≥ a x ≤ a	

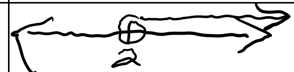
I am assuming after looking at my beautiful drawing for the graph in "open interval", you understand how to do these. Complete the chart by graphing the inequalities. We simply use number lines here because we are preparing to discuss domain, a horizontal (x) analysis of a function. **From this point on, we will use interval notation. YOU SHOULD BE ABLE TO TRANSLATE FROM ONE NOTATION TO ANOTHER!**

Discussions regarding Infinity: Please remember $\infty, -\infty$ are NOT numbers. That is why we don't see them in inequalities. They are not endpoints. -∞ : the negative is a directional meaning "coming infinitely from the left.

We must train ourselves to "read" a graph from left to right, especially when we look at a graph like x<a. The arrow on the number line will point to the left, yet we should apply a left to right sense and think "coming from the infinite left"

*Also keep in mind, even though these are number lines, we need to just keep in mind at any time, we could insert a y-axis. The number line just seems to push us to look more specifically at the horizontal.

Practice: Fill in the missing parts of the table

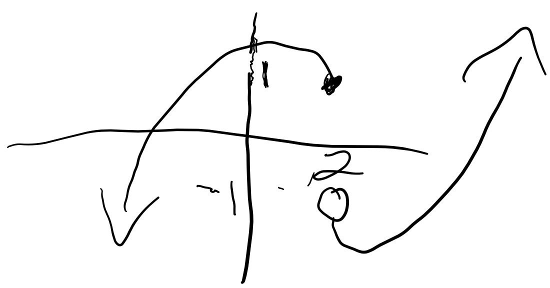
Name			
	[-8,2)		
			
		-2 ≤ x ≤ 4	
	(-∞,3]		
Open-create your own			

Continuity- By definition, a graph is **continuous** if it is a smooth unbroken curve. We have discussed this. Remember these are graphs that you could draw without picking up your pencil.

Graphs that are continuous: Sine, Cosine, a straight line, a parabola are all examples of continuous graphs, just to name a few.

Graphs that are not continuous contain some form of ***discontinuity.*** 3 Types of discontinuity are

1. **Essential**: Essential discontinuity is created by a vertical asymptote. Tangent, Cotangent, Secant, Cosecant, as well as the graph of $\frac{1}{x}$ are all examples of functions containing essential discontinuity.
2. **Removable:** Removable discontinuity is created by a hole in the graph. Take any of the above mentioned functions and put a hole anywhere on it. For now you only need to be able to see a hole and identify that as a removable discontinuity. We will discuss in a later unit how holes are created (that is, the algebra of a hole).
3. **Jump:** Jump discontinuity is what you have encountered doing piecewise graphs.



This graph has jump discontinuity at x= -2

Our goal in discussing continuity at this point is to be able to look at a graph and tell whether it is continuous or identify areas of discontinuity by naming the type and giving their x-location (as I did with jump discontinuity above.

Domain. The set of values of the independent variable(s) for which a function or relation is **defined**. Typically, this is the set of x-values that give rise to real y-values. Note: Usually **domain means domain** of **definition**, but sometimes **domain** refers to a restricted **domain**.

Think of **restricted domain** with a square root function. Graph a $y = \sqrt{x}$ on your calculator. The graph is continuous, but has no x values on the negative side of y-axis. This is a restricted domain.

With a **domain of definition**, we are thinking about the continuity of the function and algebraically what causes discontinuity. These will be the places that will cause the function to be undefined and therefore must be removed from the domain of our function.

Algebraically, we have 2 "problem children" that will create domain issues

1. We cannot divide by zero, so any value that creates a 0 in a denominator must be removed from the domain.
2. We are only working in a real number domain, so we may not have a negative number under a square root sign (nor under any radical with an even index—4^{th}, 6^{th} roots)

Examples: Find the domain of each function. State the domain using interval notation.

1. $f(x) = \sqrt{x-7}$

   The binomial x-7 is under the radical sign, so the value of x-7 can't be negative.
   So the value of x-7 must be greater than or equal to 0

   x-7 $\geq$ 0, so x $\geq$ 7—meaning x-values 7 or greater would keep this function defined
   So our final answer: Domain $[7, \infty)$

2. $f(x) = \sqrt{-3x+2}$

   Same philosophy as above. Please be careful as you solve. If you divide by a negative, the inequality turns.
   So final answer: $(-\infty, \frac{2}{3}]$

   So when radicals are involved, we will be removing entire intervals from the domain. What about when there is no radical?

3. $f(x) = \frac{x-2}{2x-5}$

Our ONLY concern here is that the denominator does not equal 0 (remember a numerator can equal 0 and when it does, the value of the fraction is 0).
The denominator, 2x-5, is a linear function, so we will only remove ONE value from the domain.

$$2x - 5 \neq 0$$
$$2x \neq 5$$
$$x \neq \frac{5}{2}$$

This is the only value to remove from the domain, so in the left to right sense of intervals, we will write our answer $(-\infty, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$

Note the $\cup$ represents union. It just means these 2 intervals together.
By leaving the intervals open and repeating the value in the next, we simply remove one value.

4. Now try $f(x) = \frac{3x+1}{3x-9}$
Final answer $(-\infty, 3) \cup (3, \infty)$

<u>But not all denominators will be linear. Let's keep going</u>

5. $f(x) = \frac{x-4}{x^2+5x+6}$

Again there is no radical, so we will only be eliminating individual values; however with there being a quadratic in the denominator, there are 2 values that must be removed.

$x^2 + 5x + 6 \neq 0$

$(x+3)(x+2) \neq 0$

$x \neq -3, x \neq -2$…think of drawing a number line with a hole at -3 and -2…then write interval

Final answer: $(-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$

6. $f(x) = \frac{x}{2x^2-5x-3}$

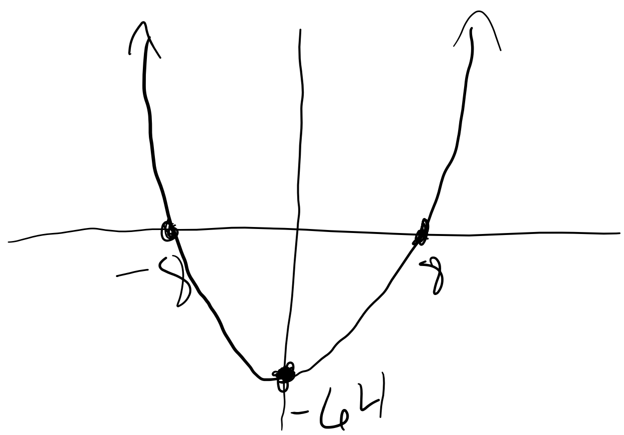
Final answer: $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 3) \cup (3, \infty)$

Notice- 1 excluded value creates 2 intervals. 2 excluded values creates 3 intervals etc

What if we mixed the problem children together?

7. $f(x) = \frac{x-1}{\sqrt{x^2-64}}$

   So we first look at the radical. What's underneath MUST be greater than 0 (cannot equal 0 because it is in the denominator).
   So what we will do is look to see where $x^2 - 64$ is positive.
   A geometric look at $x^2 - 64$ might be worthwhile as we know it's a parabola whose vertex has shifted down 64 units…not to scale (of course!)



   We know the intercepts are -8, 8 because that's where $x^2 - 64 = 0$

   Remember we are looking to find where $x^2 - 64 > 0$. The function is greater than 0 when the picture is above the x-axis. That will give the domain of f

   Final answer: $(-\infty, -8) \cup (8, \infty)$

8. Try $f(x) = \sqrt{x^2 - 100}$

   Note: without a denominator, $x^2 - 100$ can equal 0

   Final answer: $(-\infty, -10] \cup [10, \infty)$

HOMEWORK:

Use interval notation to write the domain for each function. Then graph the function on your calculator. If the function is continuous, say continuous. If it is not, give the name and location of the discontinuity.

1. $f(x) = \frac{5x+3}{x^2+8x+7}$

2. $f(x) = \frac{x-2}{x^{2-4}}$

3. $f(x) = \sqrt{9-6x}$

4. $f(x) = \sqrt[3]{5x+4}$

5. $f(x) = \frac{6x^2+2}{\sqrt{x^2-8}}$

6. $f(x) = \frac{\sqrt{9-4x}}{\sqrt{2x-1}}$

7. $f(x) = \frac{5x+4}{|5x+4|}$

8. $f(x) = \sqrt{\frac{8x-4}{x-3}}$

9. $f(x) = \sqrt[4]{9-x^2}$

10. $f(x) = (9x-2)^{\frac{3}{2}}$