Remote Lesson 4.6 OPTIONAL

Powers of Complex Numbers

In lesson 4.5, we worked with imaginary and complex numbers.

Consider the process involved in expanding $(1+i)^{2}.$ Not a difficult task. We would, rather quickly, arrive at a solution of $2i$. However, if we were to take that same binomial and raise it to a different power, say $(1+i)^{8},$ we may not be as enthusiastic about finding a solution.

One reason for learning the different forms in which a number can be written is that there may be an easier way to accomplish tasks like $(1+i)^{8}$.

Geometric Multiplication

Given complex numbers, x and w, and $x=\left[r,θ\right]and w=\left[s, φ\right]$

$xw=[rs, θ+φ]$

Example: $find xw if x=\left[4, \frac{π}{4}\right]and w=[-2, \frac{π}{6}]$

$xw=\left[4\left(-2\right), \frac{π}{4}+\frac{π}{6}\right]=[-8, \frac{5π}{12}]$

So in polar form, multiplication is fairly simple.

Example: $If, x=\left[2,\frac{π}{3}\right], findx^{4}.$

Using geometric multiplication, we again quickly arrive at an answer, namely $\left[16, \frac{4π}{3}\right].$ If we look at this problem a little more closely, it may provide some insight into to how to tackle $(1+i)^{8}$.

So using geometric multiplication we laid out the coordinate $\left[2,\frac{π}{3}\right]$ four times and multiplied the number 2 four times. In short $2^{4}$. We then added $\frac{π}{3}$ four times. In short, 4($\frac{π}{3}$).

So $[2,\frac{π}{3}]^{4}=\left[2^{4}, 4\left(\frac{π}{3}\right)\right]=\left[16, \frac{4π}{3}\right].$

DeMoivre’s Theorem

Given a complex number $z=\left[r, θ\right],$

$$z^{n}=[r^{n}, nθ]$$

So now if we go back to trying to expand $(1+i)^{8}$, our first thought should be changing $(1+i)$ to polar form where it will be easier to raise it to the eighth power.

$\left(1+i\right) in rectangular form would be \left(1,1\right).$

Changing (1,1) to polar form: $x^{2}+y^{2}=r^{2} and θ=tan^{-1}\frac{y}{x}$.

So $r=\sqrt{2} and θ=\frac{π}{4}$

Now, $[\sqrt{2}, \frac{π}{4}]^{8}=\sqrt{2}^{8}, 8\left(\frac{π}{4}\right)=[16, 2π]$

The hard work of raising the number to the 8th power is now done. The only thing left to consider is that this question was asked in complex, or $a+bi$ form so we simply have to change it back. If we use trig form for this number and evaluate it, we will be finished.

Trig form: $r\left(cosθ+isinθ\right)=16\left(\cos(2π)+isin2π\right)=16\left(1+0i\right)=16. $

Answer: $(1+i)^{8}=16$.

Most of the work here was done in converting numbers to another form; however, this is still significantly less work than using what we would call traditional methods.

Google “DeMoivre’s Thm practice worksheets” to find extra practice!