Remote Lesson 4.5 (OPTIONAL)

This would be a review of imaginary and complex numbers.

Imaginary Numbers

$i=\sqrt{-1}$

$i^{2}=-1$

Powers of *i*

$i^{1}=i$

$i^{2}=-1$

$i^{3}=i^{2}×i=-1×i=-i$

$i^{4}=i^{2}\left(i^{2}\right)=\left(-1\right)\left(-1\right)=1$

If we continue, $i^{5}=i^{4}\left(i\right)=1i=i$

Moving forward, we would watch the pattern repeat, because every $i^{4}$ is a 1, so we would see the same cycle ($i, -1, -i, 1)$ repeat. Consider when we get to $i^{8}$, we are in essence multiplying two 1s.

So if we were trying to simplify $i^{43}$, we would want to first think of that as 43 i’s next to each other. Using the pattern above if we group them in fours, every 4 i’s together would represent a 1. So what we do is group our exponents in 4s (that is $43÷4=10, with3 i^{'}s left over)$, meaning we have 10 $"i^{4}"s$. All of these have a value of 1, so $1^{10}=1.$ Now the leftovers, we have $\left(i\right)\left(i\right)\left(i\right)=i^{3}=-i$

So $i^{43}=i^{3}=-i$

Example: Simplify $i^{62}$

$62÷4=15, remainder 2. remainder 2 becomes i^{2}=-1$

Complex Number: any number of the form $a+bi$.

Operations with Complex Numbers

1. $\left(3+2i\right)+\left(-5-4i\right)=-2-2i$ (Gather like terms)
2. $\left(-1-2i\right)-\left(-3+i\right)=2-3i$ (again, gather like terms)
3. $-4i\left(2+5i\right)=-8i-20i^{2}=20-8i$ ($i^{2}=-1)$
4. $(3+i)(4$−$i)=12-3i+4i-i^{2}=13+i$
5. $\frac{4-i}{3i}$ notice the issue here is the $i$ in the denominator (a square root)

 $\frac{4-i}{3i}\left(\frac{i}{i}\right)=\frac{4i-i^{2}}{3i^{2}}=\frac{1+4i}{-3}$

1. $\frac{2+i}{4-3i}=\frac{2+i}{4-3i}(\frac{4+3i}{4+3i})$ (This is called a complex conjugate)

 =$\frac{8+6i+4i+3i^{2}}{16+12i-12i-9i^{2}}=\frac{5+10i}{25}=\frac{1+2i}{5}$

Graphing Complex Numbers in Rectangular Form: Recall, rectangular form is a coordinate plane. Since every number has a real and an imaginary part, we will simply rename the axes to real and imaginary

 Example: Graph the following in rectangular form

1. $A:2+3i$
2. $B: -1+4i$
3. $C: -4i$

So every complex number, $a+bi$, can be graphed in a rectangular form, $(a,b)$. We learned we can take rectangular coordinates and make them into polar coordinates, $[r,θ]$. Recall, $a^{2}+b^{2}=r^{2}$ and

$θ=tan^{-1}\frac{b}{a}$ . There is a fourth form in which we can write a number.

Consider: $a+bi=\left[r,θ\right] iff a=rcosθ and b=rsinθ$. (Think we would make $a+bi$ into $(a,b)$.

 Then we would change to polar

 So: $a+bi=\left(rcosθ\right)+\left(rsinθ\right)i$

 $=r(cosθ+isinθ)$ **TRIG FORM OF A COMPLEX NUMBER**

**Summarizing**

**Complex:** $a+bi$

**Rectangular:** $(a,b)$

**Polar:** $[r,θ]$

**Trig:** $r(cosθ+isinθ)$ **(this is sometimes abbreviated to** $r (cis θ)$

**Example:** Change$1-\sqrt{3}i$ to its other forms.

 Complex: $1-\sqrt{3}i$ Changing to rectangular would be

 Rectangular: $(1, -\sqrt{3})$

 Polar: $1^{2}+(-\sqrt{3})^{2}=r^{2} and θ=tan^{-1}\frac{-\sqrt{3}}{1}$

 $r=2 θ=\frac{5π}{3} remember our rect pair is in Q4$

 $\left[2, \frac{5π}{3}\right]$

 Trig: the $r and θ$ values from polar just translate into trig form

 $2(cos\frac{5π}{3}+isin\frac{5π}{3})$

 \*\*Note, if you evaluate trig form, it takes you back to complex!

Example: Change $-1+i$ to its other forms

 Rectangular: $\left(-1, 1\right)$

 Polar: $[\sqrt{2}, \frac{3π}{4}]$ (because rectangular pair is in Q2)

 Trig: $\sqrt{2}(\cos(\frac{3π}{4})+isin\frac{3π}{4})$

HW:  p. 539 3,7,11,15,19,23,27,53

                 p. 558 1,5,9,13,17,21,25

**\*\* You are responsible for material on p. 227 (1-47).  This should be review.**