Pre-Calc Area of a Triangle

Area of a Triangle

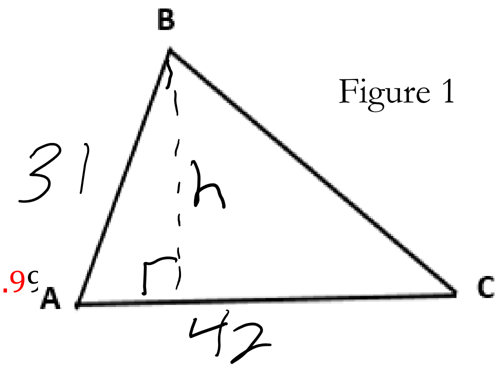
You already know this formula for the area, K of a triangle: $K = \frac{1}{2}bh$, where b is the base and h is the height.

Consider ΔABC in Figure 1.

1. AB = 31 cm, AC = 42 cm, BC = a, $m\angle A = 57°$ and the height of the triangle from vertex B to the side AC.

Find the height, h.

$$sin57° = \frac{h}{31}, h = 31sin57°, h \approx 25.9$$



2. Find the area of the triangle to the nearest cm^2.

$K = \frac{1}{2}bh$, so $K = \frac{1}{2}42(31\sin 57°)$, or if you still have your answer in your calc, $K = \frac{1}{2}42(ans)$

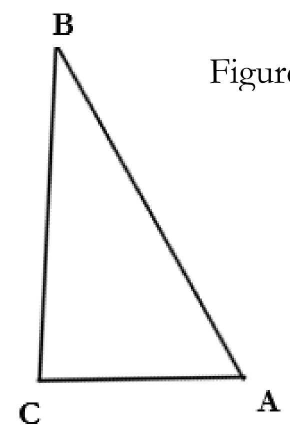
$K \approx 546\ cm^2$

3. a. Fill in the value you used for the base, b. **b =**__42____________

   b. Show how you calculated the height. **h =** ____$31sin57°$ ________

Put your answers from a. & b. into a single equation which shows how you calculated the area, **K** using the side(s) and the angle(s) of the triangle.

K = $\frac{1}{2}bcsinA$ **Notice the values given create a SAS given**



Consider the $\Delta\ ABC$ in Figure 2

In the triangle $m\angle A = 61°$ and $m\angle C = 78°$, AC = 37 in., BC=a

AB = c and the height is from vertex B to side AC.

4. Find the area of the triangle to the nearest in^2.

Notice this given is AAS. If we can find a measure to create a SAS, we can solve like above

To create SAS, we need to find a side, so law of sines will be first step.

So we know $B = 41°$. So, $\frac{37}{sin41°} = \frac{a}{sin61°}$ (we could also have used C and c instead. Either is good)

So, $a \approx 49.326\ in$ (again, keep this value in your calculator so we may use Ans feature)

We can now create a SAS having values for a, b, C

Like the formula in #1, ours will be $K = \frac{1}{2}absinC$

$$K \approx 893\ in^2$$

5. Did you need to find any of the unknown dimensions of this triangle in order to calculate the area (i.e. side c, side a, or angle B)? If yes, which dimension(s) and why?

We found B so that we could use Law of Sines. We used the Law to find a to create a SAS

6. Think about side(s) and angle(s) which were required to calculate the area of Figures 1 and 2. Within each triangle, what is the relationship of the side(s) to the angle(s)?

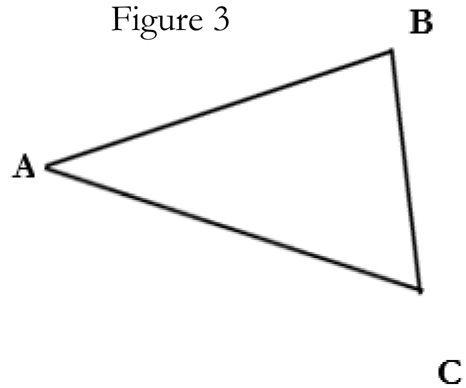
This question simply emphasizes that our goal is to create SAS

<u>Consider ΔABC in Figure 3</u>

AB = 19.2 cm, BC = 11.3 cm, AC = b, $m\angle B = 87.3°$.

$think\ c = 19.2cm\ a = 11.3cm$

Figure 3



7. If you drew the height from A to $\overline{BC}$

What is the value you would use for the base?

b = ___11.3_____________

Show how you would calculate the height.

h = $19.2 sin 87.3°$________________

8. If you drew the height form C to $\overline{AB}$

What is the value you would use for the base?

b = ___19.2___________

Show how you would calculate the height.

h = ____$11.3 sin 87.3°$__________

9. Using your answers from #7 and #8 above, write the two equations that you could use to calculate the area, **K** using the side(s) and angle(s) of the triangle.

From #7 : **K** = $\frac{1}{2} ac sin B$

From #8: **K** = $\frac{1}{2} ca sin B$

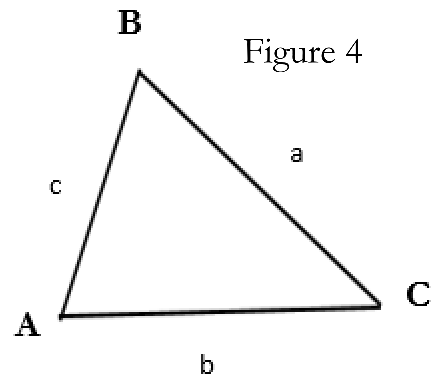
10. What do you notice about the two equations? same

Was is important where you drew the height? No, as long as it's not through the given angle

<u>Consider the ΔABC in Figure 4</u>

11. Write 3 equations that could be used to calculate the area of this triangle. One equation should use angle A, anouther should use angle B, and the third should use angle C.

Figure 4



Using angle A: **K** = $\frac{1}{2}bcsinA$

Using angle B: **K** = $\frac{1}{2}acsinB$

Using angle C: **K** = $\frac{1}{2}absinC$

Can you put into words the strategy that you would use to find the area of a triangle with an unknow height? (Discuss this with your partner) If not already given, find the measures necessary to create a SAS given, then use one of the 3 above formulas (please notice these are all the same formula, similar to the 3 versions of the Law of Cosines.)

Here is another formula you can use to find the area, K of a triangle. This formula is names for Heron who lived in Egypt from about 10-70 AD and whose proof of this oldesr in record.

Heron's Formula: $\sqrt{\boldsymbol{s(s-a)(s-b)(s-c)}}$

where a,b & c are lengths of the sides and s is the semiperimeter = $\frac{(a+b+c)}{2}$

12. Find the area of ΔABC where AB = 5 in, BC = 7in and AC = 9 in.

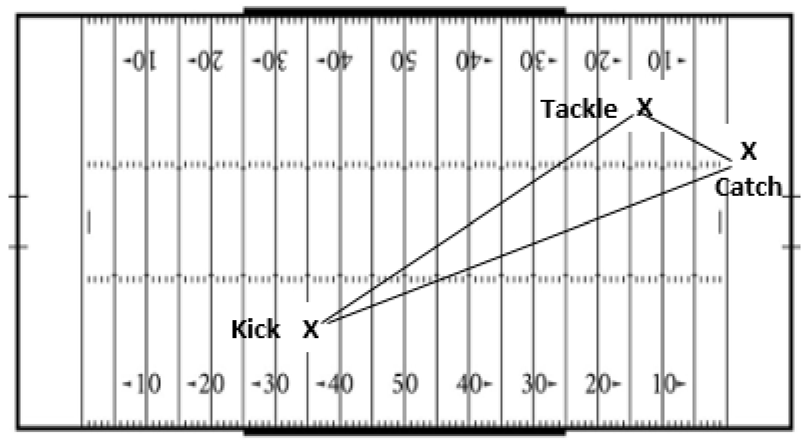
$s = \frac{5+7+9}{2} = 10.5$ so $K = \sqrt{10.5(10.5-7)(10.5-9)(10.5-5)}$

$K \approx 17\ in^2$

13. Without using Heron's formula, how else could you find the area of the triangle in #12? Do not do it, just describe how it can be done.

With a SSS given, we could use Law of Cosines to solve for any of the angles. We would then create a SAS and use one of our earlier formulas.

14. On Sunday afternoon in Charlotte, NC, the Pittburgh Steelers were playing the Carolina Panthers. At the start of the game, the Panthers kicked off from their own 35 yard line. The ball flew 73 yards and was caught by Ryan Switzer. Switzer started running at some angle from the line of the kick and ran for 12.5 yards before being tackled. The perimiter of the triangle formed by the point of the kick off, the catch and the tackle. Is 150.5 yards. What angle was formed between the line of the kick and the path of Ryan Switzer?



So our goal is to find the Catch angle. The most sensible thing to do here would be to find the remaining side by taking the perimeter and subtracting the 2 given sides. 150 – 73 – 12.5 = 64.5

So in theory we have $a = 73, b = 12.5, c = 64.5$ given to find angle C

$$\text{Law of Cosines } c^2 = a^2 + b^2 - 2abcosC$$

$$64.5^2 = 73^2 + 12.5^2 - 2(73)(12.5)\cos C$$

$$4160.25 = 5485.25 - 1825cosC$$

$$-1325 = -1825cosC$$

$$\frac{1325}{1825} = cosC$$

$$C = cos^{-1}\frac{1325}{1825}$$

$$C \approx 43.45°$$