Unit 6 Function Theory

6.1

We began by defining a function. Without calling them this, we began looking at analysis items that are answered in terms of *x*.

**Domain**: Looking at all values of x in the real number system that can be evaluated in a given function.

We begin each function with an “all reals” domain, and then search for things that would have to be excluded (zeros in denominators, negatives under even index roots). These values that need to be excluded create discontinuity in a graph (vertical asymptotes, holes, breaks). When we algebraically find a domain, we set denominators containing variables $\ne 0$ and solve to find excluded values.

If there is an even indexed root (square root, 4th root, etc), we want to make sure the function underneath the radical is set > 0 (or > 0 if the radical is in a denominator).

***Final answers for domain are written in interval notation!***

**Increasing/Decreasing Intervals**

When determining increasing and decreasing intervals, look at the function **from left to right**

Increasing, decreasing, and constant intervals should be open intervals.

**Continuity** Remember, a function is continuous if its graph is a smooth unbroken curve. Graphs that are not continuous contain *discontinuity*.

The types of discontinuity are:

*Essential discontinuity*—created by vertical asymptotes

*Removable discontinuity*—created by a hole

*Jump discontinuity*—as seen in the piecewise and greatest integer functions

***Remember, when addressing the continuity of a function, you would say the graph is continuous, or it contains essential discontinuity, removable discontinuity or jump discontinuity and YOU MUST GIVE THE LOCATION OF THE DISCONTINUITY***

**Symmetry:** Geometrically speaking, we can see y-axis symmetry and origin symmetry.

 If opposite x’s yield equal y’s, we get an even function. Even functions have y-axis symmetry.

If opposite x’s yield opposite y’s, we get an odd function. Odd functions have origin symmetry.

Remember, if the graph shifts, we use the terms ***line symmetry and point symmetry*** (and give the location of the point or line)

*Algebraically*, we can determine y-axis and origin symmetry by using the geometric definitions above and look at f(-x). By replacing all x-values with (-x), and looking at the result—

If all signs change, the function is odd—therefore origin symmetry.

If no signs change, the function is even—therefore y-axis symmetry

**When addressing symmetry, we answer *Y-axis, origin, point, line or no symmetry***

6.2

In this section, we looked at all things that need to be answered in terms of *y*.

**Range:** looking at the graph, the range is the span of y-values of the function.

 *Range is answered as an inequality*

**Boundedness:** The term bound implies a minimum or maximum *value* in a function. Remember functions with both a lower and upper bound are referred to as ***bounded***

**When addressing boundedness, we answer, *bounded above, bounded below, bounded, or not bounded.***

**Extrema:** Extrema refers to maximum, minimum, relative maximum and relative minimum values. All of these MUST be addressed when asked for extrema. Remember, to be relative, there must be intervals of function around the point. Endpoints of a function cannot be relative.

A point can be both a max and a relative max

If a function has no max, DO NOT use $\infty $. Simply say none or no max

If you use an ordered pair to name a max or min, you MUST identify it as a point. Otherwise it could be confused with an open interval!

Remember, *on a constant interval, the y-value represents both a relative max and a relative min.*

**End Behavior** this term is used to ask you to find the limits of a function as x approaches infinity or negative infinity.

Please remember to use proper limit notation!!

Know the difference between a limit of 0, a limit that does not exist, and a limit of $\infty $.

When addressing end behavior, we answer $\lim\_{x\to \infty }f\left(x\right)= and \lim\_{x\to -\infty }f\left(x\right)=$ and give the limit.

**6.3**

**Toolkit Functions**: The 12 toolkit functions were introduced. The expectation is that we know the name, equation and graph (with 3 anchor points) for each. Remember the acronym *GAELIC LOSERS* to help you remember the names of all 12 functions.

Analysis: We took the material from lessons 1 and 2 and attached it to the toolkit functions by way of analysis. 9 of the 10 steps of analysis are provided. You must remember how to answer each. YOU MUST ALSO REMEMBER TO GRAPH!!!

**6.4**

We began with piecewise functions. If it helps, change the intervals to inequalities. Remember any number in an interval must be plugged into the function so that we can establish endpoints for each segment of graph.

We reviewed graph translations: remember, if you act on x before you apply the function, then the change is horizontal. If you apply the function and then the translation, the change is vertical.

**Be able to graph horizontal and vertical changes in toolkit functions**. Establishing anchor points will help you translate functions.

**Be able to write equations for functions given changes**

 (like $given f\left(x\right)= \sqrt{x}, write the equation for the function with a horizontal shrink of 3$)

**Be able to apply translations to piecewise functions if given the graph**

**6.5**

This was a lesson on applications. Remember, the first step to success in any application problem is knowing and writing a purpose. Whether it is minimizing surface area or maximizing volume to reduce costs, know which equation is going into the calculator.

Suggestions: rather than spend time perfecting 3- dimensional figures, break the figure up into its 2-dimesional parts to calculate surface area. Remember volume is the area of the base times the height.

Be sure to connect the variables from your formula to the x’s and y’s you are typing into the calculator.

Remember, if things do not look right in your calculator, you have options: 1. Clear the memory and enter it again. 2. Go to the window and eliminate negative values as well as extremely large values.

3. Consider the domain of the function within the context of the problem. Are the x values in the window generating negative lengths?

Be sure to answer the question!

**6.6**

**Adding, subtracting, multiplying and dividing functions.**

**Composition**: f(g(x)) be able to find the composition as well as its domain. Remember finding the domain is a 3 step process found in an example in your notes.

**Inverses**: the discussion of inverses continues. Finding an inverse is a process accomplished by switching x and y and then solving for y. Remember you may be asked to state restrictions inherited from f. When this happens, one of your functions is not **one-to-one** so we restrict its domain so that the graph of the function and its inverse are reflections over the line y = x.

Also remember the connection between compositions and inverses. To algebraically verify inverses,

$$f\left(g\left(x\right)\right)=g\left(f\left(x\right)\right)=x$$